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THE APPARENT FORCES BETWEEN FINE SOLID PARTICLES TOTALLY IMMERSED IN LIQUIDS. I.

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THIS investigation was undertaken at the suggestion of Professor Milton Whitney, of the United States Agricultural Department, in order to throw light on the physical bearings of certain points in his theory of the effect of fertilizers upon the retentiveness of soils for moisture. I desire to express my obligations to him for his continual interest and valuable suggestions, and for much information and many references upon the subject of flocculation.

I would also acknowledge my indebtedness to Professor Rowland for his courtesy and assistance at all times; and to Dr. Joseph S. Ames for his continual guidance and suggestions in both theory and experiment.

§ 1. The experiments described in the first part of this paper were made in order to reproduce, if possible, some of the phenomena of flocculation under more definite and controllable conditions, while in the second part (to be published in a subsequent number of the Physical Review) an attempt is made to give a fuller theory of these phenomena than has hitherto appeared. The facts of flocculation are well known and may be summarized as follows: —

When a few drops of various solutions are added to a jar of pure water, in which clay or finely divided sand is suspended, certain marked effects occur : —

(1) If acids, lime, common salt, or almost any acid or neutral salt are added, the clay particles gather themselves into groups, — they become flocculated.

(2) When a very small trace of ammonia, caustic potash or soda, carbonate of potash or soda, or borax is added, the particles seem to attain a more finely subdivided condition than in pure water and may thus be held in suspension much longer.

(3) When an excess of these last substances is added, the same effect is produced as by the addition of acids, — flocculation is caused.

PART I. EXPERIMENTS.

§ 2. Observations on flocculation have, for the most part, been made only incidentally in experimenting upon the rate of settling of sediments in water and solutions. Professor Whitney has, however, placed a little clay-water under a powerful microscope, and upon adding a drop of very dilute hydrochloric acid has seen the particles form into closer groups. Violent currents are, of course, set up by the entry of the drop, so that individual grains can hardly be watched and little can be told of the process which has taken place except that, after the currents cease, the grains are in groups instead of separate.

§ 3. It therefore seemed interesting to try with larger bodies, whose movements could be more closely watched, whether the film of liquid, which we must suppose to be adherent to each of their surfaces — even when in so-called contact under water — could be made to change its thickness considerably by adding a slight trace of salt, and whether a sufficient force could be so called into play to move the bodies. This is, of course, a totally distinct phenomenon from the well-known interactions of floating solids, such as corks, though both are due to the same molecular forces. Experiments were accordingly begun in November, 1892, by a method suggested by Dr. Ames. A pair of large lenses, used for projecting Newton's rings in lecture experiments, were

placed one on top of the other in a vessel of pure water, and the reflected system of rings obtained. Several grams of 20 per cent salt solution were then put in. A simultaneous expansion of the rings would have meant that the glasses had moved nearer than in pure water, a contraction the reverse, but the lenses were far too heavy, and no motion could be observed even with a microscope. After several months of trial of more and more delicate modifications of this apparatus, the following method was found to answer: —

Method.

§ 4. *ABC* (Fig. 1) is a large evaporating crystal which was filled with pure water and placed under the microscope. *L* is an ordinary spectacle lens and *F* a very small fragment of a microscope cover-glass, the thinnest obtainable. A mirror reflected a

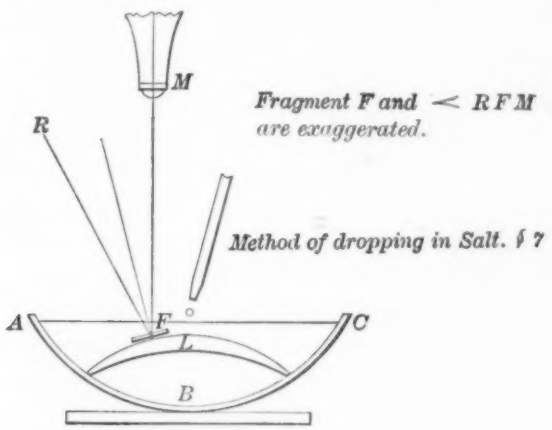


Fig. 1.

beam of sodium light *RF* on this fragment, and the Newton's rings formed between its lower surface and the lens were observed in a microscope *M*. The latter was fitted with a three-inch objective and a micrometer, whose scale was 1 mm. = 675 divisions. An average size of the fragment of cover-glass used was 400 × 600 divisions, or about one-half a square millimeter in area, weighing $\frac{1}{4}$ mg. The appearance of the rings was now carefully noted,

and their position determined with the micrometer cross-hairs. After watching carefully for half an hour or more to make sure that no accidental change was taking place in the rings, a little solution of the salt to be tried was introduced, the manner varying as the experiments gave more experience. Whether the light fragment had been attracted closer to the surface of the lens or repelled from it was now shown by the way the rings changed. The amount even of motion away from the lens could be roughly determined by the number of times the central spot changed color, the surrounding rings meanwhile shrinking. On the other hand, an expansion of the rings denoted a drawing down of the fragment, and the distance could be approximated in the same way. Further details will be found in §§ 7 to 10 below, and a consideration of the effect of change in the index of refraction and other sources of error in § 11 and § 12.

Results.

§ 5. The substances tested were:—

(1) NaCl and HCl (chiefly the former), as types of substances which tend to produce flocculation.

(2) KOH and NH_4OH, as types of those which tend to prevent it.

§ 6. The results are briefly:—

1. KOH. (*a*) The first effect of the change from pure water to very dilute KOH is to raise the fragment, off the lens, very roughly a distance of 7×10^{-5} cm. on the average.

(*b*) As the solution around the fragment grows in strength, the direction of motion is reversed, and the fragment settles back partly. This return motion did not always take place, and when it did, was usually less than the direct, so that the fragment was left further from the lens than in pure water.

(*c*) By increasing considerably the amount of KOH by further additions, the fragment was sometimes brought back to its position in pure water, and, more rarely, seemed to be drawn closer to the lens.

(*d*) By Method I., which allowed the solution about the particle of cover-glass to decrease in density as the salt first added settled

to the bottom, the fragment would be drawn down when a second drop was added, would rise again as this in turn settled, and could be made to repeat this process several times, approaching, however, a position closer to the lens as the repeated drops began to affect the strength of the solution throughout the vessel.

(*e*) By Method II., which allowed solution of definite density gradually to replace pure water, .0007 gr. per cubic centimeter was shown to be too much KOH to produce the greatest separation of lens and fragment. .0024 seems to be enough to bring them closer together than in pure water, though this limit is more uncertain.

To sum up: A trace of KOH — less than .0007 gr. per cubic centimeter — added to pure water tends to increase the thickness of the glass-liquid surface film and so separate two glass surfaces immersed in the liquid. An excess beyond this trace tends to reverse this phenomenon. (Compare effects of excess of alkali on suspended clay or sand particles § 1.)

2. NH_4OH. Few experiments were tried with this substance on account of the fumes. Of four experiments, one gave no result, one a drawing down of the fragment (flocculation), two (one of them very accurate) a result precisely like KOH.

3. NaCl. A large number of trials were made with this, most of which showed absolutely no motion of the rings. A few of the earlier experiments gave contradictory motions one way or the other. The later, more careful ones showed either no change whatever or, in one or two instances, a motion of as much as 1×10^{-5} cm. towards the lens, so small a distance that it may easily have been imagined or due to accident.

4. HCl. Tried only a very few times, usually without effect. Once there seemed to be undoubted raising of the fragment, once a slight drawing down, and once a raising followed, with increasing strength of solution, by a greater drawing down.

In a number of experiments after repeated failure with drops of NaCl and HCl, a drop of KOH on being added at once produced its characteristic effect. This indicates conclusively that the effects of KOH are not due to currents in the liquid, since the solutions were all dropped in in the same way and were, moreover, very closely of the same density and index of refraction.

Borax was tried once without result.

Details.

§ 7. The chief difficulty to be overcome was in making the change from pure water to dilute solution. At first a much larger vessel than that described in § 4 was used, and the salt was added by pouring in a few grams of 20 per cent solution at a distance from the fragment. The time taken for the solution to reach any given strength around the fragment was indeterminate, and therefore the position of the rings had to be observed with great care from the moment the salt was put in until all possibility of effect was past. This was done by making, during this time, repeated settings of the cross-hairs upon the middles of three or four successive bands, which could be done to within $\frac{1}{20}$ of the width of a band. In the course of from five minutes to an hour or more from the time of putting in the salt, there were at times considerable changes in the positions of the rings, *i.e.* a shifting in or out of $\frac{1}{4}$ or $\frac{1}{2}$ the width of a ring. There was, however, never any certainty that the changes were coincident with the arrival of the salt in the neighborhood of the fragment, and the results could never be repeated with any certainty by trying to reproduce the same conditions. It may, however, be said that observations kept up for an hour before any salt was put in rarely showed a small fraction of the change which followed its addition. The results as a whole were, however, contradictory and showed nothing.

The shallow crystal shown in Fig. 1 was now substituted for the larger vessel used before, so that a drop of solution could be put in with a rod or dropper, as in that figure, and settling to the surface of the lens, pass over the spot occupied by the fragment. It was then easy to ensure the presence of salt in the surface films between the two within a few minutes after the drop was put in by giving the lens a very slight slope and dropping in the solution a little higher up than where the small fragment F lay. The whole process was now as follows: —

METHOD I. — The lens, dish, and a number of fragments were soaked for a day in strong bichromate solution. They were then carefully washed in running water and left to soak in it over night. With a final rinsing in the morning in water filtered to

remove grit, the fragments were picked up on a glass rod and placed on the lens, care being taken to touch the latter only on the edges. The lens was then put in the dish and a final rinse given. This was sure to wash the light fragments off the lens if the surfaces were not so clean as to make very good contact, and therefore furnished a very useful though often most provoking test of the efficiency of the cleaning process. Enough filtered tap-water was now put in to cover the lens entirely, and the whole put under the microscope. The reflector was then placed so as to make the angle *RFM* as small as possible. All the fragments were then brought into the field in turn, and the one which gave the most perfect rings chosen. The position of one or more of the edges of this were then fixed with the micrometer as a check upon motion of the fragment as a whole along the lens. A rough sketch of the fragment and a few of the central rings being made, and also a note descriptive of the general appearance of the whole system of rings, the latter were carefully watched for a half hour, and, if any change occurred in their position or in the point of closest contact between the fragment and lens, the former was discarded as not trustworthy, and others successively were tried until one was found that stood the test. A drop of solution — 20 per cent in the earlier and 5 per cent in the later experiments — was now put in, as described above, and in the course of a few minutes the changes in the rings took place, whenever there were any at all. These were carefully noted, together with the time at which they took place, and were afterwards tabulated, so that the whole evidence could be seen together. Extracts from these tables are given herewith.

TABLE I.

METHOD I.

(+ means motion of rings *out*, or glass *down*.)

No.	Solution and number of drops.	Motion.		Remarks.
		Rings (number of changes).	Fragment (10^{-3} cm., up and down).	
XXIII.	KOH 20% few drops	−10 then reversed +2	−30 +6	Rings very good and complete. Size of fragment, 3 sq. mm. Both same in XXV. and XXVI.
	NaCl 20% repeated drops	No effect at all		Nos. XXIII.–XXVIII. contrast the effect of acid and alkali, no change whatever being made between the two parts of the experiment.
XXV.	HCl several	No effect		See No. XXIII.
	KOH 20% 1 or 2	Marked contraction		
XXVI.	several each of NaCl & HCl	No effect with either		See No. XXIII.
	KOH 20% 1 or 2	Marked contraction		
XXVIII.	NaCl 20% several	$+\frac{1}{2}$ −5	+1.5 −20	Very small fragment, .2 mm. in area, triangular in shape, with point of contact in one corner. About 60 of rings showing. See, also, XXIII.
	KOH 20% 3 or 4	−5	−15	
XXXV.	KOH Very dilute few drops	−1 followed by	−3 *greater* +	Washed in HCl instead of bichromate. One of very few experiments where reverse motion was greater than the original.

TABLE I. (*continued*).

METHOD I.

(+ means motion of the rings *out*, or glass *down*.)

No.	Solution and number of drops.	Motion.		Remarks.
		Rings (number of changes).	Fragment (10^{-5} cm. up and down).	
XXXI.	NH_4OH dilute	−1	−3	Showing effect of excess of alkali, as do also XXXVII. and XXXVIII. Rings clear and complete. Fragment large, about 4 sq. mm. Each time a drop of KOH entered, it drew down the fragment, which rose as the salt settled or became diffused, but not to so great a distance from the lens as before. Hence, as the solution throughout the vessel strengthened, the change each time became less, and the fragment drew continually nearer to the lens.
	1 or 2	followed by		
	more	+½	+1.5	
		−1¼	−3.7	
		(no reverse motion)		
	KOH 20% 1 or 2	+6	+18	
		followed by		
		−5	−15	
	1 or 2 more repeated	smaller + smaller −	followed by each time	
XXXVII.	KOH 20% 1 drop	−1½	−4.5	Washed in very strong bichromate solution for 3 hours and then in running water for 2, and obtained very clear and steady rings.
	several more	no effect		
	more yet	+1.5	+4.5	
		followed by		
		−¼	−3.8	
	twice again	each time same as the last growing less		
XXXVIII.	KOH 5% 1 drop	−2.5	−7.5	First experiment after four months' absence. Very much like the last, except that the fragment does not seem to have risen between the last drops.
		followed by		
		+1.7	+5.0	
	more	+½	+1.5	
	more	no effect		

§ 8. After the preliminary methods had been given up, all the experiments, except six with KOH, were made substantially as above. Not one single instance of a contraction of the rings following the addition of NaCl was observed by this method, and most of these experiments were never even tabulated. Of six that are included in the table, three show no motion whatever, and three a very slight spreading of the rings, the greatest amount being one-half the width of the central spot, *i.e.* corresponding to an approximation of the two surfaces by 1.5×10^{-5} cm. This lack of effect is easily explained on the theory that NaCl would, if possible, produce a drawing together of the surfaces, as it flocculates clay; but they could actually move closer only in a few accidental cases, since the weight of the fragment, of itself, pressed it into contact with the lens, owing to the care which was taken to have the surfaces perfectly clean. With KOH, on the other hand, within a very few minutes after the drop went in, the rings would almost invariably draw in towards the center, the latter changing color about four times on the average. This change occupied about one-half minute, and was then often followed by an immediate reversal of the motion of the rings. This reverse motion was sometimes rapid, at times quite gradual, but very rarely did it indicate a motion beyond the point from which the fragment originally started. Sometimes it took a dozen drops to bring the fragment as close to the lens as it would go, and in the meanwhile each drop, as it entered, would draw it down, and in five or ten minutes it would commence to rise once more as the drop flowed past and solution around the fragment weakened, only to be again drawn down a little further by the entry of the next drop, thus indicating that for each degree of the strength of solution there was a tendency to a film of liquid of definite thickness between the glasses.

The trials with HCl and NH_4OH need not be dwelt upon, as, where any results followed, they were like those with NaCl and KOH. The most interesting experiments by this method were those where a number of drops of NaCl (or in two experiments HCl) were added and produced no effect, or, in a few instances, a slight approximation of the glasses; and then, without in any

manner changing the position of the little fragment or any other condition of the experiment, a drop of KOH was put in, and found at once to produce its characteristic separation of the glasses. It was thus shown very conclusively that the effects with this salt were in no way accidental, but depended on a specific difference between it and the two others, since the drops were all put in in the same manner, and were carefully made of very nearly the same refractive index. Moreover, by reversing the process and adding the alkali first, the same difference was shown.

§ 9. METHOD II. — To show more conclusively that the effect of KOH was not due to disturbance on account of currents in the liquid, and to gain, if possible, a more definite idea of the density of the solution which accompanied the phenomena, the following more elaborate method was used a few times, only with KOH :—

The lens was supported near the top of a much deeper vessel, into the bottom of which liquid could be introduced by a siphon from a glass reservoir. The latter was supported on a stand of variable height, so that the rapidity of the flow through the siphon could be regulated or even reversed if necessary. The vessel containing the lens being placed under the microscope, the siphon was started and the vessel filled. The flow was then stopped, and the best fragment, as to rings and stability, was selected by trial as before, and the flow of water being again started, was watched for half an hour or more to test the effect of the mere current upwards due to the siphon, which was run at a considerably greater rate than that afterwards used. If, as was usually the case, the fragment was found to be so stable that the rings were in no degree affected by this much more rapid flow, the reservoir was filled with water to a known mark and a measured quantity of the KOH solution stirred into it. Now starting the flow at a very slow rate, the salt solution of known concentration very gradually rose in the vessel under the microscope, the pure water flowing off at the top. The fragment was always so placed that it would be directly under the point of overflow, so that the alkalinity of the liquid in which it lay could be easily determined by touching test paper to this overflow. In this manner the

moment when the liquid at the point in question changed from pure water to solution of known strength could be ascertained very closely. Its very remarkable coincidence with the time of the changes in the rings in the few trials made by this method, served to preclude any doubt that one was the cause of the other, while the almost imperceptible strength of current prevented the phenomena from being ascribed to such mechanical disturbance. These experiments gave, further, the rough values of the respective strengths of solution which caused the greatest separation of the two surfaces and a closer approximation than in pure water. These were given in § 6.

TABLE II.

METHOD II. (BY SIPHON.)

(+ means motion of the rings *out*, or fragment *down*.)

No.	Grams KOH per cu. cm. siphoned in.	Motion.		Remarks.
		Rings (number of changes).	Fragment (10^{-3} cm. up and down).	
XLII.	.0024	+2.5	+7.5	No contraction at all; fragment seems suddenly to have passed from the pure water into solution dense enough to show effects of excess of alkali. See section 10 and Fig. 2.
XLVI.	.0008	−2	−6	Washed in fresh bichromate. Center of rings shifted a very little, otherwise experiment was very good. After the first part of the experiment the .0008 solution was run off and siphon filled with the stronger solution.
		followed by		
		+1.5	+4.5	
	.0015	slight further +		
XLVII.	.0007	−1.5	−4.5	Made to test the effect of acidity or alkalinity of surface before experiment. Everything precisely the same as in XLII. except that between the two the lens and fragments were soaked in strong KOH and then in running water.
		followed by		
		+5	+1.5	

§ 10. The way in which observations of changes in the rings were usually noted is shown by the sketches in Fig. 2, which are copied from my note-book, Experiment No. XLII. The shape of the fragment was a trifle more irregular, but roughly a triangle in which $AB = .89$ mm., $AC = 1.42$, $BC = 1.54$. The same ring is made heavy throughout and serves to fix the amount and direction of the change in the whole system. The siphon ran at varying

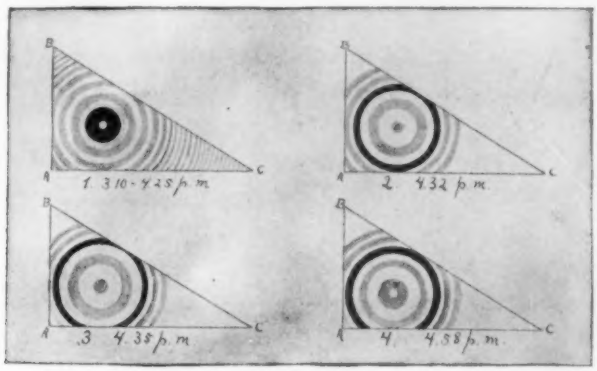


Fig. 2.

rates with pure water (Method II. was used) from 2.45 P.M. to 3.25, and was then started with KOH solution whose strength was .0024 grams per cubic centimeter. The change noted in sketch 2 began at 4.25. An expansion from the center of two rings is shown, as also that the stability of the fragment was not very good, since it tilted over a very little towards AC during the experiment.

In the earlier experiments distilled water was used, but it was found that that available was a little greasy, and in all the later trials filtered tap-water was used for the final washing and the experiment itself.

Sources of Error.

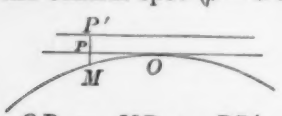
§ 11. CHANGE IN THE INDEX OF REFRACTION. — Let the point P (Fig. 3), in a plane tangent at O to a sphere of radius R, be at a distance $MP = e$, from the sphere, measured perpendicular

to the plane; and let $OP = \rho$. Then if P' is a point in a parallel plane at a distance p above the tangent one, $t = p + e = MP'$.

Hence $$\rho^2 = 2Re = 2R(t-p) = \left(\frac{n}{2\mu}\sec. r - 2m\right)\lambda R, \qquad (1)$$

where λ is the wave-length in air; r the angle of refraction through the film of liquid, and μ the index of refraction of the latter; and where $m = \frac{p}{\lambda}$, *i.e.* m is the distance p expressed in wave-lengths in air. n is the number of the ring of radius ρ from the center and is, of course, even for a dark band and odd for a light one.

The point observed in the experiments was the number of times the central spot ($\rho = 0$ approximately) changed through a complete cycle, *i.e.* from n to $n-2$, n being always counted from the ultimate black spot which would occupy the center were the lens and fragment in perfect contact. If, then, n changes from n to $n+\delta$ when salt is put in and the index of the dilute solution thus formed is μ_1, then the condition that it is the central spot which is being observed gives



$OP=p.$ $MP=e$ $PP'=p.$
$MP'=t.$
Fig. 3.

$$\frac{n}{2\mu}\sec r - 2m = \frac{n+\delta}{2\mu_1}\sec r - 2m_1 = 0,$$

or $$m_1 = \frac{\mu}{\mu_1}m + \frac{\sec r \cdot \delta}{4\mu_1}.$$

Therefore, if

$$\frac{\mu}{\mu_1}m + \frac{\sec r \cdot \delta}{4\mu_1} > m\,;\ \textit{i.e.}\ \frac{\sec r \cdot \delta}{4} > m(\mu_1 - \mu),$$

m_1 is greater than m, *i.e.* the particle has moved away from the lens. In order, then, that the change δ shall be wholly accounted for by the change in the index of refraction, m, the distance of the fragment from the lens at starting must be at least as great as

$$m = \frac{\sec r \cdot \delta}{4(\mu_1 - \mu)}.$$

I give below the requisite values of m for various values of δ, the number of times the center changed color. The data are for 5 per cent and 20 per cent solutions, the strength of the *drops* put in, which was very much diluted before reaching the fragment, and for $r = 0$, which makes m a minimum.

	μ_1.	$\mu_1 - \mu$.	m		
			$\delta = 1$.	$\delta = 2$.	$\delta = 4$.
NaCl 5 %	1.340	0.006	41	83	167
20 %	1.369	0.035	7	14	29
KOH 5 %	1.341	0.007	35	71	142
20 %	1.360	0.026	9	19	39
NH_4OH	—	negative	∞	∞	∞

The average change in the experiments due to a few drops of 5 per cent KOH was a contraction of two rings ($\delta = 4$), and to account for it wholly we should have to suppose a distance of 142 wave-lengths or $\frac{1}{10}$ mm. to separate the two glass surfaces supposed to be in contact at starting. Further, the change in the rings with NaCl should have been greater than with KOH; and NH_4OH should have produced the opposite change to what it did. The indices for such dilute solutions are not given in Landolt and Börnstein's tables, but were calculated from refraction equivalents. They are at 18° and μ for water is taken as 1.334.

§ 12. Osmotic Pressure. — A more important source of error was suggested by Dr. Ames, who thought there might be a very strong force tending to lift the fragment due to imperfect diffusion of the salt into the interstice between the two glasses. A method of attacking the question was found in an extension of an equation given by J. J. Thomson.[1] The Lagrangian function of the molecules of salt (supposed to behave like those of a perfect gas) in the volume of a solution and also in the surface films bounding it, is there given. Here two similar terms were added for the volume of liquid between the glasses and the two glass surfaces bounding thereon, the density of the salt being supposed to differ there from

[1] Application of Dynamics to Physics and Chemistry, pp. 190, ff.

its value in the open liquid immediately around. The glass surfaces are for simplicity assumed plane and parallel and separated by a distance x. If, then, H is the Lagrangian function of the salt molecules in the interstice and the liquid immediately around the fragment, $\frac{\delta H}{\delta x}=F$ is the force tending to separate the two. To determine F completely would require a knowledge of the rate at which the solutions become equalized in strength as x increases under the action of the force. A minimum value, not far from the true one, may be found by assuming that the only change is that due to the flow into the interstice, as the particle rises, of enough strong solution from without to fill the increase of volume.

Let v_1 represent the volume of liquid between the fragment and the lens; $\frac{1}{2}S_1$, the area of the lower face of the fragment, *i.e.* S_1 the total area of glass surface bounding the volume v_1; t_1 the thickness of the film on this surface and T_1 its surface tension, ρ_1 and σ_1 the densities of salt in v_1 and S_1 respectively; and $\eta_1=v_1\rho_1$ and $\xi_1=\sigma_1 S_1 t_1$ the total weights of salt in each. Let the same characters without subscripts denote similar quantities in the liquid immediately around. J. J. Thomson's equation for H is (adding two necessary terms in the subscript quantities):—

$$(1)\qquad H=\eta R\theta\log\frac{v\rho_0}{\eta}+\eta\{f(\theta)-\omega_1\}-ST$$
$$+\eta_1 R\theta\log\frac{v_1\rho_0}{\eta_1}+\eta_1\{f(\theta)-\omega_1\}-S_1T_1$$
$$+\xi R\theta\log\frac{St\rho_0}{\xi}+\xi\{f(\theta)-\omega_1\}$$
$$+\xi_1 R\theta\log\frac{S_1t_1\rho_0}{\xi_1}+\xi_1\{f(\theta)-\omega_1\};$$

where θ is the temperature and R an analogous quantity to the same symbol in Thermodynamics, *i.e.* $R\theta\rho=p$, the osmotic pressure; ω_1 and ρ_0 are constants of integration not depending on x. Remembering that

$$(2)\qquad \frac{\delta\eta}{\delta x}+\frac{\delta\eta_1}{\delta x}+\frac{\delta\xi}{\delta x}+\frac{\delta\xi_1}{\delta x}=0;$$

since the total quantity of salt is not a function of x, we get

$$F=\frac{\delta H}{\delta x}=R\theta\left[\left\{\frac{\delta\eta_1}{\delta x}+\frac{\delta\xi_1}{\delta x}\right\}\log\frac{\rho}{\rho_1}-\tfrac{1}{2}S_1(\rho-\rho_1)\right].$$

For complete solution we need an equation giving

$$\frac{\delta\eta_1}{\delta x}+\frac{\delta\xi_1}{dx},$$

the rate of inflow of salt as the fragment rises, but we can find a minimum by assuming that no diffusion takes place during the very short time occupied by the change in the rings, and no weak solution flows out to be replaced by stronger. Then

$$\frac{\delta\eta_1}{\delta x}+\frac{\delta\xi_1}{\delta x}=\rho\frac{\delta v_1}{\delta x}=\tfrac{1}{2}\rho S_1,$$

since $\frac{1}{2}S_1$ is the area and x the height of the space v_1. Substituting we get the minimum value of the force,

$$F=\frac{R\theta}{2}\left[S_1\left\{\rho\left(\log\frac{\rho}{\rho_1}-1\right)+\rho_1\right\}\right].$$

Remembering that $p=R\theta\rho$, we have

$$F'=\frac{S_1}{2}\left\{p\left(\log\frac{\rho}{\rho_1}-1\right)+p_1\right\},$$

where S_1 is the area of glass bounding on the interstice, *i.e.* twice the area of the lower surface of the little piece of cover-slip; ρ_1 is the density of salt in the interstice and ρ in the liquid outside; and p_1 and p are the pressures of a number of hydrogen molecules per unit volume equal to the number of salt molecules in densities given. The assumption that the salt in dilute solution follows Avogadro's law is to be noted. From data in Everett's "C. G. S. System of Units," $p=4\times10^8\rho$ and $p_1=4\times10^8\rho_1$. Hence

$$F'=2\times10^8S_1\left\{\rho\left(\log\frac{\rho}{\rho_1}-1\right)+p_1\right\}\text{ dynes,}$$

or

$$4\times10^8\rho\left\{\frac{\rho_1}{\rho}-\left(1+\log\frac{\rho_1}{\rho}\right)\right\}$$

dynes per square millimeter of the under surface of the glass fragment. In the following table F' is given for various ratios of

the density in the interstice to that in the liquid around. ρ is taken as .00075, an average value in the experiments by Method II., where alone the density was definitely known.

F' FOR KOH.

$\frac{\rho_1}{\rho}$.	F'.	$\frac{\rho_1}{\rho}$.	F'.	$\frac{\rho_1}{\rho}$	F'.
0.00	∞	0.10	4200	0.70	180
0.01	10830	0.30	1500	0.90	15.7
0.05	6150	0.50	520	1.00	00

In order to compare this with the force downward due to the weight of the fragment, a whole cover-slip was weighed and its diameter measured, and was found to weigh .46 mg. per square millimeter, or to be subject to a downward force of 2.94 dynes per square millimeter in water, which is insignificant compared to F'.

Considering the first effect with KOH alone, *i.e.* the rise of the fragment as the drop of solution reaches it, this force would abundantly explain the action, since the fragment ought to rise until the densities above and below it became equal, and then, the force having ceased to exist, drop back. The explanation is insufficient because: —

(1) Were this force alone acting, a second drop should again cause the fragment to rise, since the solution beneath it would again be more dilute than that outside, but in the majority of cases the reverse was the case and a second drop, if it produced any effect, usually caused a fall of the fragment, which is quite in accord with the fact that excess of alkali flocculates.

(2) So far as could be found,[1] NaCl, in ordinary cases, diffuses less rapidly than KOH, and if this holds for very dilute solutions and capillary openings, we should expect a greater disparity in the strength inside the interstice and without in the case of NaCl, and consequently a greater lifting force, whereas no lifting at all was observed. The lifting force for equal ratio of the strength inside and out is with NaCl 95 per cent of that with KOH.

[1] Ostwald's Solutions, trans. by Muir, p. 134.

While, in the absence of sufficient data with regard to diffusion in capillary spaces, some may prefer to consider the experiments as largely explained by the osmotic force, it seems to the writer more natural to suppose, considering both (1) and (2), that the effects are the result of two sets of forces both tending to raise the fragment in the case of dilute KOH, and, on the other hand, almost exactly counterbalancing each other in the case of NaCl and stronger KOH, the first set being the molecular forces which act in flocculation, and the other those which bring about diffusion.

§ 13. To sum up: The effects summarized in § 6 are a reproduction of those observed in the flocculation of clay particles, as may be seen from the following comparison: —

Results of the Experiments.	*Flocculation.*
1. The effect of very dilute alkalies, represented by KOH, is to raise the glass fragment away from the lens.	1. Fine solid particles are held further apart in very dilute alkaline solutions than in pure water.
2. As the KOH solution became stronger the fragment returned to its original position, and at times seemed drawn closer to the lens.	2. More concentrated alkaline solutions reverse this effect and cause flocculation.
3. If the glass surfaces were closely in contact at starting, very little effect should have been expected from substances tending to draw them closer. They could only act by bending the fragment. This lack of effect was found with NaCl, in marked contrast with the strong effect of KOH.	3. The effect of NaCl and the other substances of its class is to produce a closer grouping of the particles of clay than in pure water.

The writer hopes to extend these qualitative results to a wider range of substances and, if possible, obtain quantitative observations connecting the thickness of the adherent films separating the glasses with the density of the solution.

THE DISTRIBUTION OF ENERGY IN THE SPECTRUM OF THE GLOW-LAMP.

(PRELIMINARY MEASUREMENTS.)

BY EDWARD L. NICHOLS.

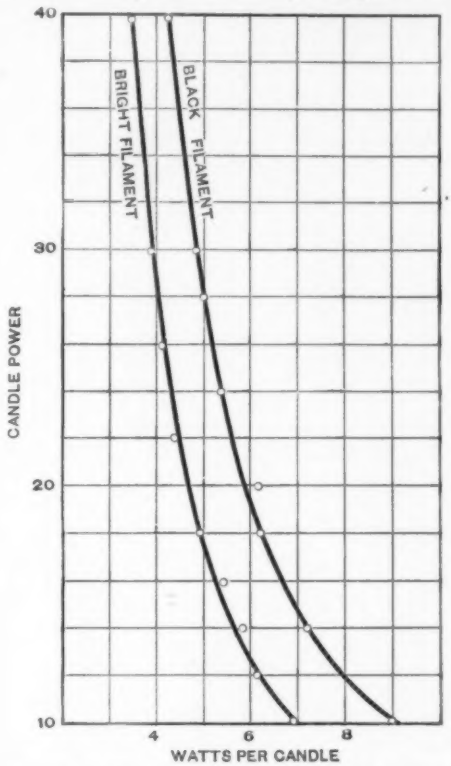
I.

THE glow-lamp affords an interesting subject of study. In the first place, it is possible by varying the current in the filament to secure a very wide range in the temperature of the radiating surface, extending upwards to temperatures considerably higher than those of ordinary luminous gas flames, and to maintain more constant the condition of incandescence during the progress of each set of measurements than is possible in the case of any other source of light. In the second place, the study of the spectrum of the incandescent lamp affords an answer to certain questions of considerable practical importance.

In 1885, Messrs. Siemens and Halske, of Berlin published the results of measurements for the purpose of showing the superiority of the silver-gray surface obtained by treating filaments of glow-lamps by bringing the same to incandescence in an atmosphere consisting of volatile hydro-carbon. In the following year Mr. Mortimer Evans[1] described comparisons of the radiation from bright and black incandescent lamp filaments in which the superiority of the former was very clearly demonstrated. Evans selected two filaments of similar size and structure, one of which he subjected to a surface deposition obtained by flashing in an atmosphere of ordinary coal gas; the result being a hard coating of carbon of the color of lampblack. The other filament was flashed in an atmosphere of volatilized hydro-carbons, by which means the usual silver-gray surface was obtained. The two fila-

[1] Evans, Proceedings of the Royal Society, Feb. 18, 1886.

ments were then mounted and exhausted to the ten-thousandth of an atmosphere, and characteristic curves were made, showing, (1) the relationship between the efficiency of the lamps, expressed in watts per candle, and candle power (see Fig. 1); (2) the relation between the total watts consumed in each lamp and the candle power. These curves show a very much greater amount of light for the energy expended in the case of the filament with bright surface.

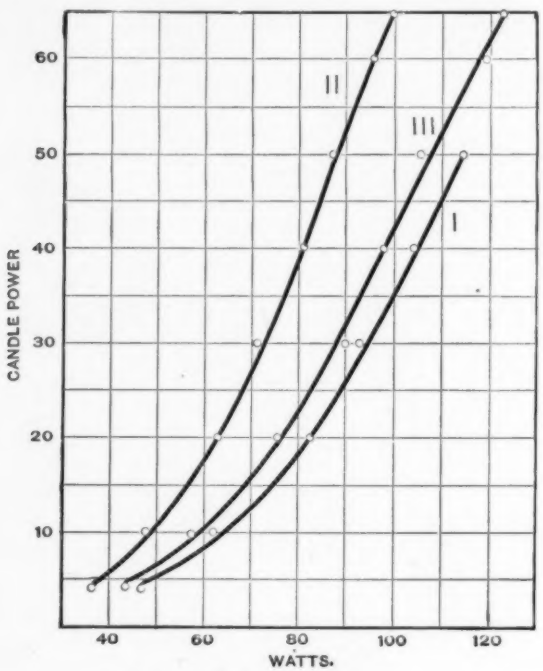


Efficiency of Black and Bright Filaments (after Evans).

Fig. 1.

Subsequently, two filaments of close firm structure and of equal size were obtained which possessed the usual dull, dead black color resulting from carbonization without treatment. The characteristic curve of one of these was obtained after mounting the same in the usual manner and securing a good vacuum, after which it was removed from the lamp and was flashed in hydro-carbon vapor. After exhaustion it was again tested, measurements showing marked increase in efficiency; then it was removed from the bulb for a second time and flashed in coal gas, by which process a new surface of the color of lampblack was obtained. Having been remounted in this final condition and thoroughly exhausted for the third time, another characteristic curve was obtained. This curve was found to coincide quite closely with that of the original untreated filament. These three curves are given in Fig. 2.

Mr. Evans' experiments seem to leave no question as to the fact that the bright surface obtained by flashing in hydro-carbon vapor possesses properties which fit it peculiarly for the production of light. Evans noted the fact that the lamp bulbs containing the black filaments were much hotter than those which surrounded filaments with a silver-gray surface, and he remarks: "I have little



I. An Untreated Filament.
II. Same, after treatment (bright).
III. Same, after treatment (black).
(After Evans.)

Fig. 2.

doubt that the loss of efficiency when black was due to the energy supplied being radiated in large quantities as heat waves from the blackened surfaces, which these surfaces when bright would not emit."

That the *total* radiating power of carbon in the form of lamp-black is very much greater than that of the silver-gray graphitic form obtained by flashing in hydro-carbons has been shown by

H. S. Weber in a paper read before the Electrical Congress in Frankfort in 1891.[1] Weber, who tested thirty-three varieties of glow-lamp filaments, found for the black carbon a coefficient of total emission lying between 0.0000169 and 0.0000174, and for treated filaments with bright surface, coefficients lying between 0.0000127 and 0.0000132.

During the winter of 1891–92, Mr. John C. Shedd[2] made a large number of comparisons of commercial lamps with treated and untreated filaments, for the purpose of further testing the conclusions reached by Evans. His method consisted in studying the visible spectrum of such lamps by means of the horizontal-slit photometer. The lamps were maintained for this purpose at various degrees of incandescence.

Mr. Shedd's results were in the main confirmatory of the higher efficiency of bright carbons, but they indicate clearly that the efficiency of the incandescent lamp is dependent upon other factors than the character and radiating power of the surface of the filament. He found, for example, that the vacuum especially has much to do with the relationship between the energy developed in the lamp and the candle-power.

His comparisons between glow-lamps, at different stages of incandescence, and the gas-flame, bring out two facts which are of interest in the comparative study of treated and untreated filaments.

(1) Spectro-photometric measurements with lamps of untreated filaments of vegetable fiber showed that the light was of the same color as that from gas (in the argand burner) when the lamp was being operated at an efficiency of 6.8 watts per candle. All efficiencies lower than this showed relatively higher intensities in the red, while all efficiencies above 6.8 showed a light which was bluer than that of the gas. The relative intensities of the various wave-lengths in the light emanating from such filaments were found to vary quite rapidly as the incandescence rose. Such a lamp, for which the ratio $\frac{\text{glow-lamp}}{\text{gas-flame}}$ for the region of the D line was unity, gave for wave-length 0.4 μ about 1.6.

[1] Weber, Verhandlungen des Internationalen Elektrotechniker-Congresses zu Frankfurt am Main, 1891, II., p. 49. See also Physical Review, Vol. II.

[2] Thesis: in Library of Cornell University, 1892.

(2) In the case of treated filaments with bright surface, on the other hand, equality with gas-light as regards whiteness was reached only at somewhat higher efficiencies, viz. between 4 and 6 watts per candle. The changes in the color of the light from such lamps appeared, moreover, to be much less marked than in the case of filaments with black surface. The variation in quality, indeed, was frequently scarcely noticeable when the lamp was brought up from 7 to 4 watts per candle. It would appear from Mr. Shedd's measurements that in the case of black filaments the increase in the intensity of short wave-lengths with temperature was much more rapid than in the case of the longer wave-lengths; whereas with bright filaments all the wave-lengths of the visible spectrum gained in intensity at very nearly the same rate.

II.

A study of the results obtained by Mr. Shedd made it evident to me that the influence of the vacuum of the glow-lamp was so great as to render all comparisons unsatisfactory between different lamps in which the pressure was an unknown factor. In the hope of overcoming this difficulty I had some lamps constructed, through the kindness of a friend engaged in the manufacture of glow-lamps, by means of which this difficulty could be obviated.

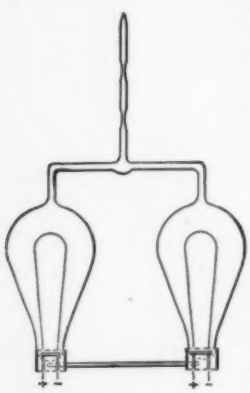
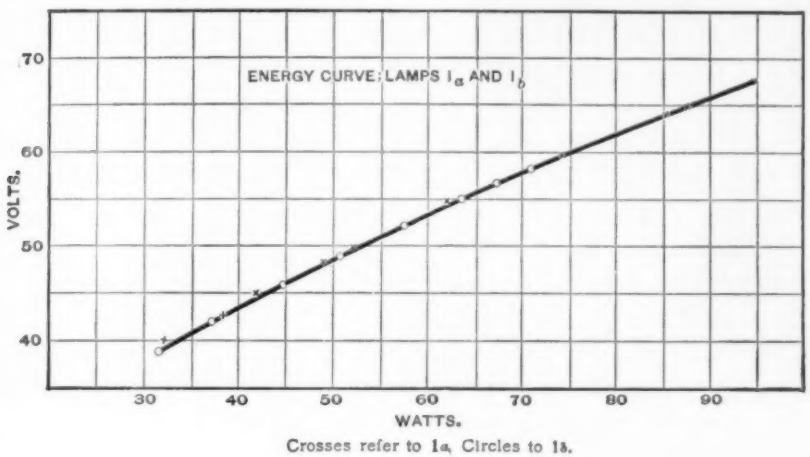


Fig. 3.

Two lamp filaments as nearly identical as possible were taken. These were brought to identity as regards resistance by the usual method of flashing in hydro-carbon vapor; one of them was then given a coat of black carbon by smoking. They were mounted in bulbs of the usual form, the two being connected by a glass tube as shown in Fig. 3. The pair of lamps thus connected were placed upon a pump together, and were exhausted in the usual manner, after which they were sealed. These lamps could be brought to incandescence separately; but since there was permanent communication be-

tween the interiors of the two bulbs, whatever changes in vacuum occurred in the one would be shared by the other lamp.

Upon measuring the candle-power, voltage, and current, in the case of these two lamps throughout a wide range of incandescence and platting the characteristic curves, it was found that the difference in efficiency as expressed by watts per candle was very great indeed. The efficiency of the smoked filament was extraordinarily low. Inspection of the results obtained made the cause of this difference obvious, the fact being that the mantle of carbon obtained by smoking was of extremely high electrical resistance. The result was that when the same current was sent through



Crosses refer to I_a, Circles to I_b.

Fig. 4.

the two lamps, the total watts generated in each were identical (see Fig. 4); but since the radiating surface of the smoked filament was very largely increased, the temperature of the lamp was greatly lowered. The consequence was that the black lamp required a higher voltage and a correspondingly greater amount of energy to bring it to a given candle-power than did the bright filament, which was without the superficial non-conducting layer of carbon. It seems probable that something of the same kind may have occurred to vitiate the experiments made by Mr. Evans, although, doubtless, the non-conducting film in the case of his lamps was much thinner than in the pair of lamps now

under consideration. Although it was impracticable to compare these lamps with one another by the mere determination of the candle-power and the watts per candle, they afforded excellent material for the study of the laws of radiation of these two widely different carbon surfaces. I accordingly subjected them to detailed measurements as regards the distribution of energy in the infra-red of the spectrum through a very wide range of temperatures, making use, for this purpose, of a method nearly the same as that described by E. F. Nichols in the first volume of this Review.[1] Indeed, in many of these measurements Mr. Nichols

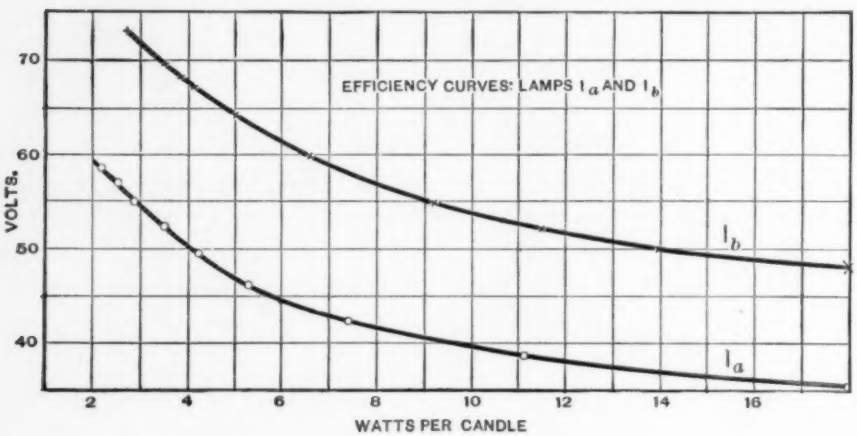


Fig. 5.

did me great service, bringing to bear upon what was in many respects an operation of unusual delicacy the skill attained by long practice in a similar research.

III.

The apparatus used in the study of the spectrum of these lamps was that described by Mr. Nichols in the paper just cited. It consists of a spectrometer, in place of the eyepiece of which is inserted a linear thermopile of antimony and bismuth, the edge of which had been filed down until only a very narrow strip was exposed to radiation. Dispersion was obtained by means of a

[1] Nichols, PHYSICAL REVIEW, Vol. I., p. 1, 1893.

prism of carbon bi-sulphide contained between glass walls. This substance was selected because there is reason to believe that it obeys more closely than any other available material Cauchy's law for dispersion.

The use of the spectrometer with glass lenses, and of a prism the walls of which are of glass, in the study of the distribution of energy in the spectrum, is open to the serious objection that both glass and carbon bi-sulphide exert selective absorption upon the rays of light which pass through them. The only substitute for a prism and lenses would be a concave grating, or a plane grating used in connection with a concave mirror. The feebleness of the spectra produced by means of the grating offers in itself a serious hindrance to its use in bolometric work. The theory of the diffraction grating, as given by Rowland, is, moreover, conclusive as regards the unfitness of ruled gratings for the determination of energy in the spectrum, and it had been previously shown experimentally by Paschen that the exploration of the spectrum of incandescent lamps by means of the grating and bolometer led to results which were full of the most serious errors. No more striking verification of the theory can be given than the curves exhibited in Paschen's paper. So serious, indeed, are the vagaries of the diffraction grating in this respect that it would seem as though all work involving absolute measurements of the distribution of energy in the spectrum, in which diffraction has been made use of, must be discarded and the work done over by other methods.

As regards the fitness of the material used in the apparatus with which the experiments to be described were made, it may be noted that Ångstrom has studied the absorption of carbon bi-sulphide in the infra-red and has found it to be transparent as far as wave-length 3.0μ, beyond which lie two strong absorption bands. Since, however, my measurements do not extend beyond 3.0μ, there seems to be every reason to assume that no serious errors will be introduced by the passage of the rays under investigation through this medium.

Ernest Nichols, in the article just cited, has studied the diathermancy of glass. His curve for the percentage of light trans-

mitted by a plate of glass (see PHYSICAL REVIEW, Vol. I, p. 8) indicates great uniformity in this material, up to the longest wave-lengths to be considered. The diminution of light due to the interposition of the specimen of glass which he tested, is scarcely greater than the losses by reflection at the two faces of the plate, and while there is evidence of very slight decrease in the transmitting power as we pass from 2.0μ to 3.0μ, the change cannot amount to more than one or two per cent. Under these circumstances, it seems quite safe to assume that no serious errors will result from selective absorption of the rays of the infra-red spectrum of glow-lamps, either by the glass of the lenses and the walls of the prism, or by the contents of the prism itself.

The most serious objection to the use of the bi-sulphide prism lies in the fact that the dispersion is very small for the longer wave-lengths. The shortening of the spectrum in the infra-red is such that measurements beyond wave-lengths 3 μ are of questionable value.

The choice of the thermopile, the width of the face of which was something more than half a millimeter, instead of a more strictly linear instrument such as the bolometer, is open to criticism. In point of fact, the thermopile showed itself to be so much more sensitive than any bolometer which Ernest Nichols and the writer were able to construct, that we were led to adopt it in spite of this objection. It seemed quite certain, moreover, that there was but little of interest to be found in the spectrum of glow-lamps beyond the regions which it was possible to explore with the thermopile, so that it did not seem worth while to expend a great deal of time in the construction of a bolometer.

Our experience appears to differ from that of others who have worked in this field; but it may not be without interest to state that we both tried bolometers constructed of the platinum cross-hair wire, used by Snow in his exploration of the bright line spectra of the alkalies, — an investigation which would seem to demand the very highest delicacy; also bolometric strips of iron, nickel, aluminium, etc. We were not able within the time at our disposal to construct a bolometer, the sensitiveness of which for the purposes of our measurements was greater than one-twentieth of that of the linear thermopile.

The galvanometer used in these studies was the one briefly described by Ernest Nichols in his article already cited. It was built in the instrument shop of the department by F. C. Fowler, but the moving parts were made by Mr. Nichols, to whom also I owe the very careful study of the bi-sulphide prism.

IV.

Mr. Nichols, in his paper, has described briefly the calibration of the bi-sulphide prism. Since, in work of the kind under consideration, the accuracy of this calibration is of the utmost importance for the determination of wave-lengths in the infra-red, it may be well to give some additional details. The containing vessel of the prism consisted of a block of cast iron bored through so as to give a cylindrical opening about 4 cm. in diameter. The glass faces of the prism were cemented over this opening, forming an angle of almost exactly 60°. Since the refractive index of carbon bi-sulphide changes greatly with the temperature, it was necessary to have an accurate knowledge of the temperature of the contents of the prism at the time when each measurement was made. The method finally adopted for this purpose was as follows: —

At the back of the prism a small pocket was constructed, one side of which was formed by the iron wall of the prism itself. Within this a delicate thermometer was inserted, the bulb of which was surrounded by water.

The dark room, within which the spectrometer was mounted, was an inside room of the laboratory, containing only one opening, that, viz., through which the rays from the incandescent lamp which it was our purpose to study reached the collimator lens of the spectrometer. The temperature changes in this room were very slow, and a comparison of the readings of the thermometer with those of a similar instrument, the bulb of which was placed within the liquid of the prism itself, showed that the difference of temperature within and without, through the very slight range to which both would be subjected, was negligible. Having satisfied ourselves that the indications of the thermometer in the water

pocket at the back of the prism might be taken, the other thermometer was withdrawn and the prism was sealed up. Readings of the remaining thermometer were made before and after each set of readings.

The prism was standardized for a temperature of 23.5° C., a reading which represented very nearly the mean temperature of the room, and all observations were reduced to this temperature. The entire range to which the prism was subjected in the course of the measurements was about 3°. The indices of refraction of the prism were determined by Mr. Nichols with great care for temperatures lying within the above-mentioned range, the wave-lengths selected for this purpose being those of the C and F lines of Fraunhofer. The results of this calibration, which are given in Table I. of Mr. Nichols' paper, are in close agreement with similar determinations made by Wüllner.

The method of calibration consisted in determining with all possible accuracy the Cauchy equation of the prism from the indices of refraction at the standard temperature for wave-lengths corresponding to the lines A, B, C, and *b* of Fraunhofer. By means of the equation thus obtained a curve was platted extending to the wave-lengths in the infra-red which we wished to measure. This method of determining the longer waves is less satisfactory in some respects than that made use of by Mouton and afterwards by Rubens, but it was believed that the errors thus introduced would not be important in comparison with those which result from other sources.

V.

Briefly stated, the method of experimenting was as follows: —

The thermopile already described was placed in closed circuit with the galvanometer, the latter instrument having been sensitized to a degree found by previous trials to suffice for the exploration of the region to be tested. The spectrometer was then set to a circle reading corresponding, at normal temperature, to the wave-length in question. The thermometer was then read, the time noted, and the inner room was closed, not to be entered again until the measurements for that region of the spectrum had been completed.

The thermal disturbances produced in the course of these preliminary adjustments lasted in uncontrollable intensity for about thirty minutes, after which time it became possible to bring the galvanometer needle to a position such as to render the scale visible to an observer at the telescope. The drift, still very rapid, would then diminish in the course of ten minutes more to a state of comparative repose, such that readings for the ballistic throw could be obtained. The next step was to close the lamp circuit and to read volts and amperes. The galvanometer was then brought to the proper part of the scale, and preliminary calibrating throws were taken. For the purpose of calibration a subsidiary coil was used. This coil was mounted permanently about twenty centimeters behind the needles. It could be thrown into shunt circuit with a compensated resistance through which flowed a very steady current from a storage battery. The current was measured from time to time by means of an accurate mil-ammeter. The swing due to the closing of the circuit of the subsidiary coil gave the relative sensitiveness of the galvanometer. Such calibrations were made before and after every reading of the thermopile throws, and thus all the determinations were rendered reducible to a common scale, although the figure of merit of the galvanometer varied greatly in the course of the experiments.

After the completion of the preliminary calibration, the radiation measurements were begun, sets of five throws being taken for each condition of the lamp, which was changed stepwise between each set until the entire range to be covered had been included.

When all the readings to be made for the region of the spectrum to which the thermopile was exposed had been completed, the inner room was entered, the thermometer was read, and the time noted. The spectroscope was then set to a new wave-length, and after thermal equilibrium had again established itself, the operations above described were repeated. In this way the entire infrared spectrum between 0.8 μ and 3.0 μ was explored.

The data thus obtained were reduced in the following manner:—

(1) All deflections were brought to a common scale by reference to the continually repeated calibration throws.

(2) Wave-lengths were determined by correcting the values cor-

responding to the circle readings of the spectroscope for temperature. These corrections were obtained graphically from calibration curves platted to a large scale.

(3) The deflections were corrected for the width of the face of the thermopile. This was a most important correction, since, owing to the fact that the spectrum was prismatic instead of being normal, the surface of the pile received radiation from a larger and larger number of waves as the wave-lengths increased.

To be comparable one with another, in different parts of the spectrum, the deflections should be those obtained by exposure of

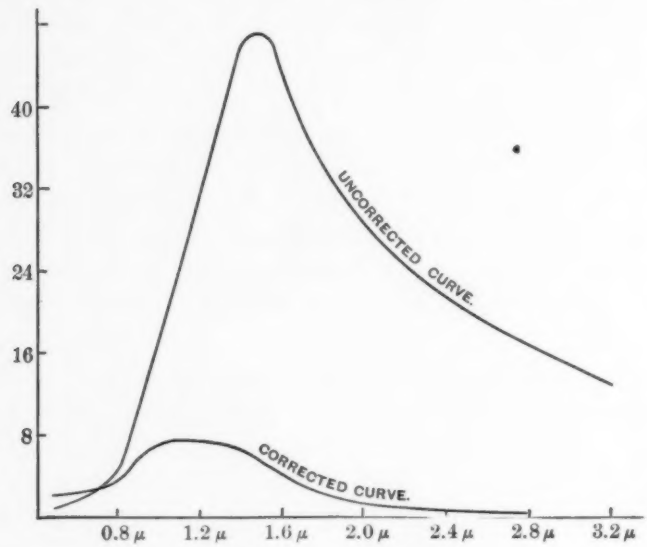


Fig. 6.

a surface varying in width with its position, in such a manner as always to receive radiation from a region of unit breadth, measured in wave-lengths. This correction it was possible to apply with satisfactory approximation up to wave-length 3 μ. How important this correction was will appear from Fig. 6, in which a corrected and uncorrected curve are platted to a common scale. In this diagram the width of the thermopile is taken as normal for the region of the D line.

The two lamps described in the second section of this paper

were measured throughout a range of temperatures from the red heat upwards to a condition of incandescence beyond which they could not be carried without rapid deterioration. The corrected results are given in the following tables: —

TABLE I.

SPECTRUM OF GLOW-LAMP 1_b (BLACK FILAMENT).

Wave lengths.	Intensities at					
	35 watts.	45 watts.	55 watts.	65 watts.	75 watts.	85 watts.
0.8 μ	0.360	0.744	1.140	1.490	1.820	2.220
1.0	0.859	1.450	2.050	2.700	3.410	4.180
1.2	1.560	2.350	3.170	4.040	5.010	6.090
1.4	2.280	3.150	3.970	4.730	5.460	6.160
1.6	1.560	2.050	2.590	3.050	3.490	3.830
1.8	0.859	1.140	1.360	1.630	1.860	2.080
2.0	0.490	0.656	0.793	0.923	1.050	1.160
2.4	0.148	0.195	0.240	0.284	0.325	0.362
2.8	0.0465	0.0613	0.0768	0.0910	0.106	0.194

TABLE II.

SPECTRUM OF GLOW-LAMP 1_a (BRIGHT FILAMENT).

Wave-lengths.	Intensities at								
	75 watts.	65 watts.	55 watts.	45 watts.	35 watts.	25 watts.	15 watts.	10 watts.	5 watts.
0.7 μ	2.22	1.66	1.08	0.626	0.240	0.107	—	—	—
0.8 μ	3.30	2.62	1.90	1.260	0.744	0.464	0.248	0.224	—
0.9	5.45	4.22	3.13	2.270	1.480	0.900	0.372	0.306	—
1.0	6.43	5.16	3.17	2.970	2.040	1.300	0.461	0.312	0.167
1.2	6.84	5.84	4.75	3.790	2.790	1.74	0.732	0.372	0.313
1.4	6.27	5.50	4.72	3.880	3.050	2.06	0.799	0.599	0.419
1.6	4.40	3.86	3.27	2.680	2.030	1.33	0.572	0.418	0.257
1.8	2.20	2.02	1.78	1.480	—	0.813	0.361	0.281	0.179
2.0	1.31	1.20	1.07	0.908	—	0.523	0.255	0.199	0.115
2.4	0.64	0.527	0.467	0.400	0.333	0.258	0.147	0.125	0.118
2.8	0.28	0.259	0.233	0.204	0.169	0.133	0.077	0.0633	0.0469

Of these two sets of measurements, those upon the lamp with the bright filament were the more complete, being carried down to temperatures much below the red heat. The results may be treated graphically in either of two ways:—

(1) By the construction of curves with intensities as ordinates

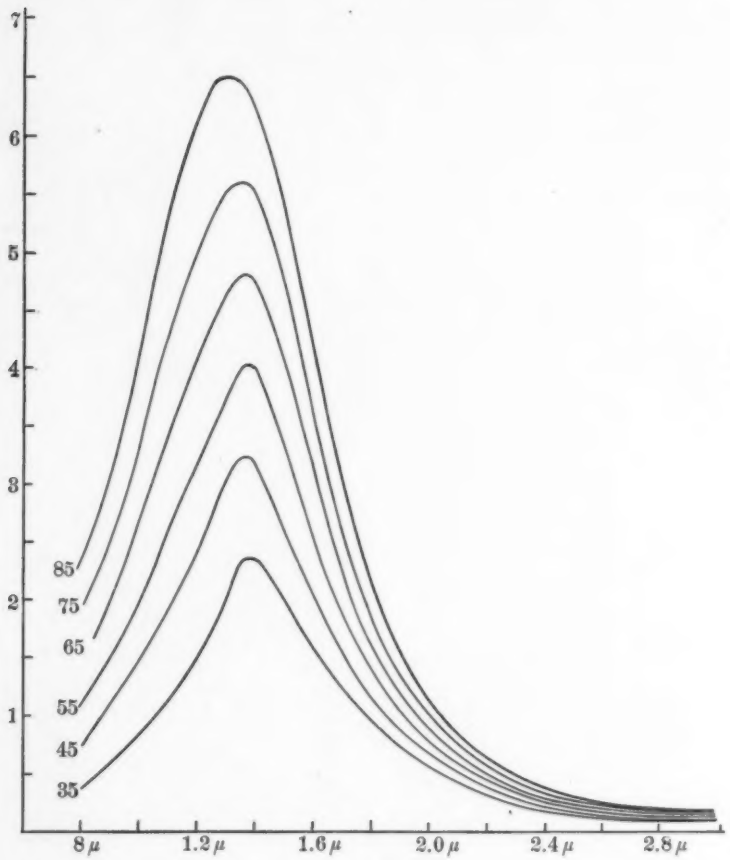


Fig. 7.

and electrical activity expressed in watts as abscissas, each curve representing a separate wave-length.

(2) By platting intensities as ordinates, and wave-lengths as abscissas.

Curves obtained by the former method show the rise in intensity

for each wave-length with increase in the energy expended in the lamp. These curves are very similar in form to those for the radiation of platinum as a function of the temperature, as measured by the writer[1] and also by Violle.

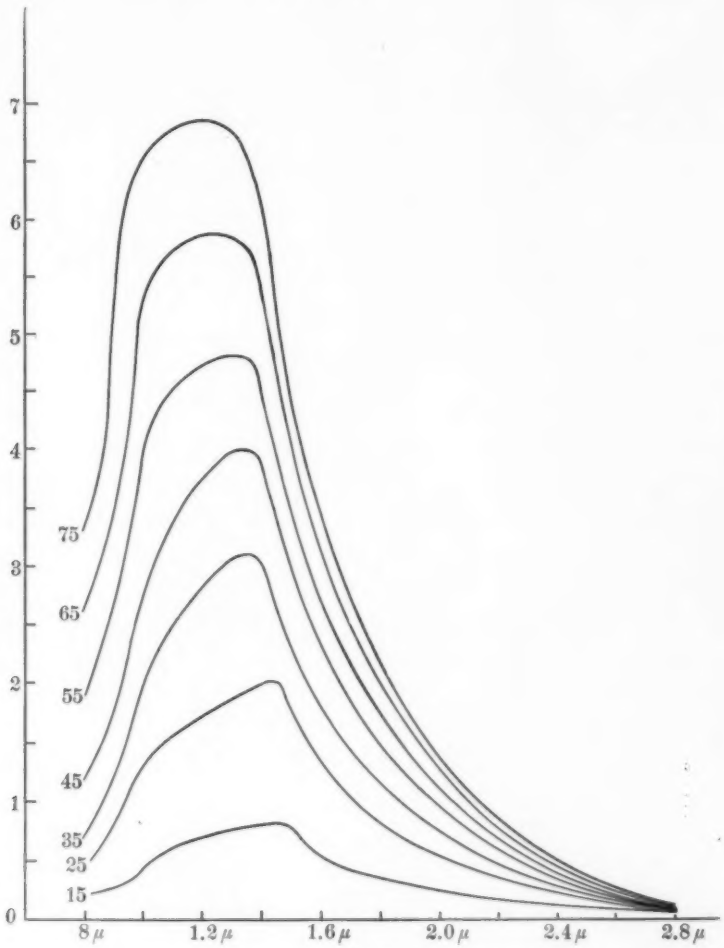


Fig. 8.

The curves of the second type are shown in Fig. 7, which shows the distribution of energy in the spectrum of the lamp with lamp-black-coated filament (I_b), and in Fig. 8, which gives similar curves

[1] Nichols, Ueber das von glühendem Platinum ausgestrahlte Licht. Göttingen, 1879.

for the lamp with bright filament (I_a).[1] It will be seen that the distribution is very different in the two cases, the maximum in the case of the latter being nearer the visible spectrum. Were both sets of curves extended to the shorter wave-lengths, it would unquestionably be found that the area of the portions lying within the range of the luminous rays is larger in the case of bright filaments, a result which would afford a complete explanation of the superiority of such filaments in the production of light, as shown by Evans in the experiments already described.

Similar measurements to the above were made upon a commercial lamp with untreated filament bamboo. The lamp in question is the one used as a reference light in the experiments of E. F. Nichols upon absorption in the infra-red.[2] It will be seen, by reference to his curves, that the distribution in the spectrum of this lamp was in every respect similar to that in the spectrum of lamp I_b.

The complete investigation of the problem with which this present paper deals would involve the direct measurement of the temperatures of the radiating surface. In such an investigation, to which the experiments here described are preliminary, the writer is now engaged. The results are to be given in a subsequent article.

[1] The only measurements in any way comparable with these, so far as the writer is aware, are those of Abney and Festing. Their curves are, however, platted with scale readings instead of wave-lengths as abscissas, so that it is difficult to make any exact comparison.

[2] E. F. Nichols, *l.c.*

THE INFLUENCE OF HEAT AND THE ELECTRIC CURRENT UPON YOUNG'S MODULUS FOR A PIANO WIRE.

By Mary Chilton Noyes.

Object of the Investigation.

MOST of those who have investigated experimentally the effect of change of temperature upon the elasticity of metals have found that the elasticity decreases as the temperature rises. Wertheim's[1] results, however, indicate an increase of elasticity for iron and steel with rise of temperature, the elasticity reaching a maximum somewhere between 100° and 200° C. He was such a careful experimenter, and the increase observed was in some cases so considerable, that it seems improbable that these results are altogether in error. One object of the present investigation was to see if any indication could be found of a maximum value for the modulus between 100° and 200°.

Many of those who have experimented on this subject have used only a few temperatures, and few have gone to temperatures beyond 100°. There have also been considerable variations in the results. It seemed desirable to obtain the modulus for a series of temperatures extending nearly to 200°, and to secure the various temperatures by a method not previously employed in investigating the subject.

General Plan of the Experiments.

The different temperatures desired have been obtained by means of an electric current from a storage battery. The wire to be tested was mounted horizontally on a comparator, and carefully adjusted so as to be parallel with the lines of the com-

[1] Ann. de Chim. and Phys., Ser. 3, 12, 385.

parator bed. It was placed inside a glass tube of about $2\frac{1}{2}$ cm. diameter for protection against air currents. On the outside of this tube was wound a helix of German silver wire. In some of the determinations the test wire was maintained at the desired temperature by means of a current through this helix, while in others a current was passed through the piano wire itself. It was thought that if the magnetization had any effect upon the elasticity, it would be different in the two cases. If the two methods of heating gave similar results, as it was expected they would, it was supposed that the magnetic effects could be regarded as negligible. It will be seen later that the results obtained did not agree with this supposition.

Two openings were made in the glass tube, through which the positions of two marks on the wire were observed by microscopes. When the heating was accomplished by means of a current through the helix, the temperature was determined by a thermometer placed inside of the glass tube, but so arranged as not to touch it. Currents of air were avoided by stopping the ends of the tube with cotton. When the current passed through the test wire itself the temperature was determined by the expansion of the wire, and also by measuring its electrical resistance.

The wires tested were piano wires of two sizes, the larger 0.40 mm. in diameter, and the smaller 0.26 mm.

Preliminary Experiments.

The micrometer heads of the microscopes were first calibrated. It was found that for the left microscope 4000 divisions were equal to one mm., and for the right microscope 4013 divisions. In the calculations both have been taken as giving readings of $\frac{1}{4000}$ mm.

The temperature coefficient for electrical resistance was next determined for the larger wire. The wire was coiled on a piece of glass tubing covered with silk, and then immersed in a vessel of oil. The resistance was determined for temperatures ranging from near 0° to about 180°. Several determinations were made, and the curve showing the relation of resistance to temperature plotted; but the curves all showed quite a sharp bend at about

75°. This was found to be due to the fact that the viscosity of the oil at lower temperatures caused the temperature of different parts of the bath to be unlike. Greater care in keeping it thoroughly stirred secured a curve which was very nearly a straight line.

The following results were finally obtained: —

Resistance at 0° C.	= 3.680.
Average change in R for 1°	= 0.01428.
Coefficient of R compared with R_0	= 0.00388.
Resistance at 20°	= 3.967.
Coefficient of R compared with R_{20}	= 0.00360.

The observations were carried only a little above 100°, but as all the other results above that point had indicated a constant temperature coefficient, it was thought that no error would be introduced by not carrying the determinations further.

The diameter of the wire was measured with one of the microscopes, but it was found impossible to focus on the exact edge so as to make the determination certain. The diameter was therefore afterwards measured with a micrometer wire gauge and found to be 0.40 mm. The density, as computed from this diameter and the weight of a measured length, was found to be 7.785. The density was also determined by weighing a coil of the wire in water from which the air had been exhausted by an air pump, and also by weighing it in water in which the coil had been boiled so as to free it from air. The average density found from three determinations made in this way was 7.775. As this differed by only about one-tenth of one per cent from the result obtained from the micrometer measurement, that measurement was taken as the correct one.

Considerable difficulty was found in securing a uniform temperature in different parts of the glass tube which contained the wire. The helix was made at first of German silver wire, and at the lower temperatures seemed to give quite satisfactory results. But in some of the preliminary experiments a strong current was sent through the helix and it blackened unevenly, after which it was found that there were large variations in temperature between

those parts of the tube where the wire was darkest and lightest. The helix was removed and a fresh one substituted, with the expectation that by using currents of only moderate strength it could be kept in good condition. It began to turn black in places, however, even when the temperature inside the tube was only about 150°. The attempt was made to oxidize it uniformly by sending a strong current through the parts that were least blackened, but this was found impossible. An iron coil was then tried and proved to be much more satisfactory.

The coefficient of linear expansion with rise of temperature was determined for each kind of wire.

The results obtained were as follows:—

LARGE WIRE.

	I.	II.
Length of wire	=891.181,	889.105 mm.
Average elongation for 1° =	43.57,	43.31 Mic. Div.
Coefficient of expansion	= 0.0000122,	0.00001218.

Average coefficient of expansion = 0.00001219.

SMALL WIRE.

Length of wire	=893.0 mm.
Average elongation for 1° =	42.28 Mic. Div.
Coefficient of expansion	= 0.00001183.

Arrangement of Apparatus.

After a number of preliminary experiments to determine the best arrangement of apparatus, that shown in Fig. 1 was finally adopted.

AB is a movable iron bed of the comparator, resting on the support *CD*. *GH* is the glass tube, resting on wooden blocks, and having openings at *E* and *F* through which the readings were taken by the microscopes *M* and *N*. The wire was fastened firmly to a wheel at *R*, passed over a grooved block of wood at *K*, through the glass tube *GH*, over the wheel *L*, to the weights suspended at *Q*. The wheel *L* was taken from an Atwood

machine; the axle of the large wheel rests at each end between two smaller wheels. A pasteboard tube protected the portion of the wire between the glass tube and the wheel from currents of air. Only that portion of the wire between *E* and *F*

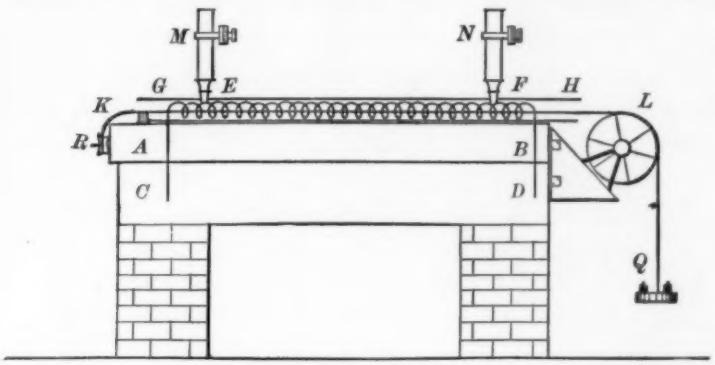


Fig. 1.

was tested, but when the portion outside the glass tube was unprotected, drafts of air striking this outside portion had so much effect in changing the temperature of the part between *E* and *F* that no reliable results could be obtained. The wires for the

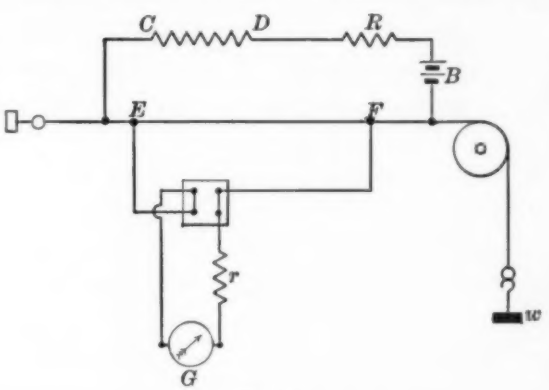


Fig. 2.

current, and for measuring the resistance, were attached near *G* and *H*.

The plan of connections for making the resistance measurements is given in Fig. 2, the fall of potential method being used.

The portion *EF* of the wire to be tested, was in circuit with the battery *B*, the resistance *R* being adjusted so as to give the desired temperature. For some of the experiments in which a weaker current was desired, the circuit was closed through *R*, and *EF* connected in parallel with such a portion of *R* as would give the desired strength of current. *EF* was connected in parallel with the galvanometer *G*. The resistance box *r*, in the galvanometer circuit, was for the purpose of reducing the deflection, and also to make the resistance so large that the current in *EF* would not be appreciably diminished by closing the galvanometer circuit. *CD* is a compensated resistance in the circuit *EF*. By putting it in parallel with the galvanometer instead of *EF*, and comparing the deflections, the resistance of *EF* could be determined.

Considerable time was spent in finding what weights would give the most concordant results. Those used at first were too small to keep the wire horizontal; when weights were added to determine the elongation, the wire was drawn out of focus and the microscopes could not be accurately set. It was finally found that by using 2.6 kg. for the smallest weight, and increasing it twice by 0.4 kg. the wire could be kept in focus, and the elongations were such as could be read accurately and conveniently; a greater elongation would have required setting the microscopes nearer the edge of the field of view than could have been done with precision.

For most of the determinations two sets of readings were taken, each set consisting of the changes in length produced by adding 0.4 kg. twice, and removing the same weight twice, thus making the result the average of eight determinations. When the wire was heated, care was taken to see that it returned very closely to the original length when the weights were removed; if there was much variation it was concluded that the temperature had varied and the set was discarded. In many cases, however, it was found that the average results of these discarded sets would be almost identical with those obtained with constant temperature, since if the change was regular the effect upon the elongation with increasing weights would be balanced by the

effect upon the contraction with decreasing weights. As it was impossible to tell when the change was regular and when it was irregular, the only safe way seemed to be to discard such sets of readings entirely. Since a change of one-tenth of a degree produced an elongation which was about the same as the difference in the elongation for 0.4 kg. made by a change of 40°, a very steady temperature was necessary to secure reliable results.

For each measurement the microscope was set twice and the average reading taken ; if the readings differed from one another by as much as one or two divisions, more observations were made. The weights were generally placed in position a minute or two before the readings were taken. A comparison of readings made as quickly as possible after placing the weights, with those obtained after waiting a minute or two, showed more consistent results in the latter case. The aim was to make the conditions of determining the modulus as nearly identical in all cases as possible, so that the results at different temperatures might be strictly comparable.

In order to bring it to the "state of ease," each wire was stretched for 24 hours or longer by a weight somewhat greater than the maximum weight to be used in determining the modulus, before any observations were made.

The formula used in computing the modulus is

$$M = \frac{P \cdot L}{Q \cdot l},$$

where

P = weight in kg.,
L = length of wire,
Q = cross-section,
l = elongation.

In computing M the length has been expressed in millimeters and the cross-section in square millimeters. Since the value of g in Ithaca is 980, the results given in the following tables must be multiplied by 980×10^6 in order to reduce to C.G.S. units.

EXPERIMENT 1.

DATA FOR CURVE A, FIG. 3.

Number.	T	Elongation for 0.4 kg.	No. of obs.	M
1	20.0	485.15	16	23388
2	42.5	490.35	14	23143
3	55.9	492.35	8	23046
4	65.9	495.50	4	22900
5	83.0	500.38	8	22676
6	103.5	513.15	10	22112
7	122.0	519.25	4	21852
8	147.0	539.50	9	21032
9	164.5	525.10	6	21609
10	200.0	550.75	6	20603

$L = 891.181$ mm. $Q = 0.12566$ sq. mm.

In the above table are given the elongations, expressed in divisions of the micrometer, for a weight of 0.4 kg. in the case of the first wire tested. The number of observations used in obtaining each result is also given, together with the corresponding values of the modulus. The corresponding curve is marked *A* in Fig. 3.

The higher temperatures were obtained, in this case, by means of a current through the helix surrounding the wire. Before any observations were made, the wire had been heated and cooled a great many times in making the preliminary experiments and in determining the coefficient of expansion. The annealing effect of this heating is probably the reason why the modulus is higher than was found afterwards for other pieces of the same kind of wire.

The curve shows plainly a decrease of elasticity with increasing temperature, and that the decrease is nearly proportional to the change in temperature.

EXPERIMENT 2.

Another piece of wire was taken from the same coil and heated as the first had been, by a current through the helix around the

glass tube. The points marked with a cross near curve *B*, in Fig. 3, are those corresponding to the results of this experiment. The line marked *C* in the same figure is the same as *B*, only values for the modulus at nearly the same temperatures have been

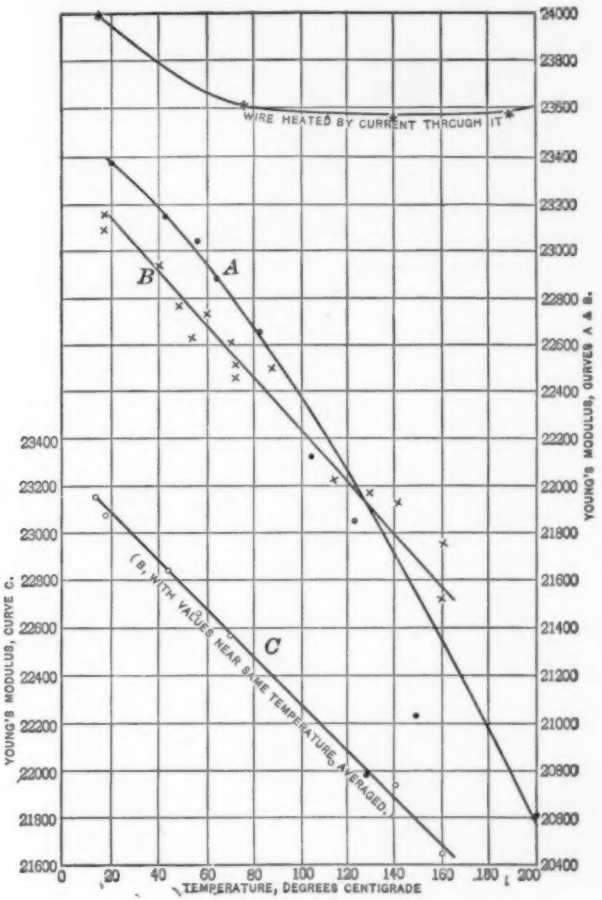


Fig. 3.

averaged instead of being plotted separately. The line is nearly straight, showing a decrease of elasticity which is proportional to the increase of temperature. The inclination of the line is very nearly the same as that of curve *A* up to about 70°; the slight change in the inclination of the first curve above that

temperature is probably due to lack of accuracy in the determinations at the higher temperatures.

The first measurements in this experiment were made after the wire had been stretched so as to bring it to the "state of ease," but before it had been heated at all. The final result at about the same temperature showed an increase of about three-tenths of one per cent, doubtless due to the annealing effect of heating the wire.

EXPERIMENT 3.

The same wire was used as in the previous experiment, but this time it was heated by a current through the wire itself. The table gives the elongations and the corresponding moduli.

DATA FOR POINTS MARKED WITH A STAR IN FIG. 3.

Number.	T	Elongation for 0.4 kg.	No. of obs.	M
1	30.9	484.88	8	23344
2	16.5	471.9	10	23986
3	78.0	479.5	12	23606
4	140.0	480.38	8	23562
5	190.0	480.33	6	23565

$L = 889.105$. $Q = 0.12566$ sq. mm.

The first current, which heated the wire to about 31° as determined by the expansion, gave a higher modulus than had been obtained at the temperature of the room even after the annealing effect of a temperature of 160°. After the first determination, the current was increased so much that the wire broke. After it had again cooled, a still higher value of the modulus was found. It was then heated to different degrees by a current through it. The value of the modulus was somewhat less than when the second result in the table was found, but was still nearly two per cent greater than the greatest value that had been found before the current had been sent through it at all. The points marked with a star in Fig. 3 are those corresponding to this experiment.

The results seemed to indicate that the effect upon the elasticity due to heating the wire by a current around it and through it are very different, and that in the latter case the elasticity is considerably greater than it would be if the wire were heated to the same temperature by the former method.

Experiment 4.

The results obtained in the previous experiment were so unexpected that it seemed desirable to verify the phenomena. Another piece of the same kind of wire was accordingly adjusted. After the modulus had been determined at the temperature of the room, a weak current was sent through the wire; the modulus was found to be a little higher than before. The wire was then allowed to cool, and the modulus determined at the temperature of the room; it was found to be still higher. Currents of varying strength were then sent through the wire, and the modulus determined at the different temperatures. The values found were all very nearly alike.

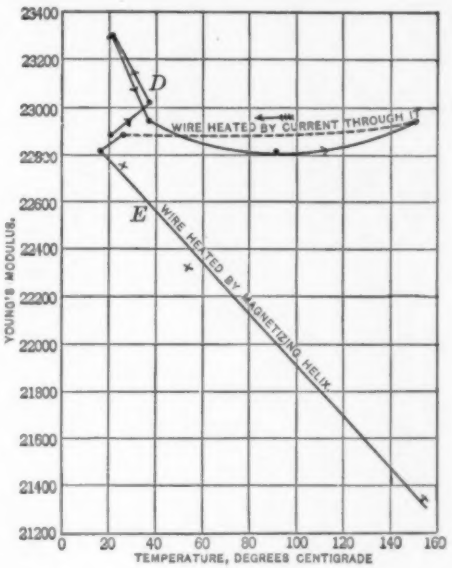


Fig. 4.

The curve marked *D* in Fig. 4 represents the results of this experiment. The temperatures are quite uncertain, as the adjustments for measuring the electrical resistance were incomplete, and it was found very difficult to determine the higher temperatures by the elongation. The change in length was rapid when the current was started, and was so large that the mark was carried a distance several times as great as the field of view.

There was no doubt, however, that there was a wide range in the temperature, and that the highest was 150° C. or above.

The results seem to indicate that the magnetizing effect of a current through the wire increases the modulus by an amount which very nearly balances the decrease that would be produced by the same increase of temperature secured by a current around the wire.

In order to see whether there had been such a permanent change in the molecular condition of the wire as would make the

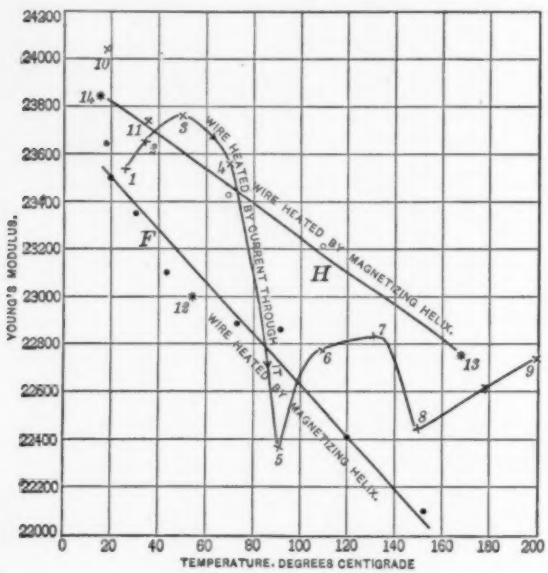


Fig. 5.

effect of heating it by a current around it different from what had been found in the other pieces, the modulus was determined at several temperatures secured by a current through the helix. The resulting curve, marked *E*, is shown in the same figure with the other results for the sake of comparison. The modulus is seen to decrease with increasing temperature, and the inclination of the line is very nearly the same that had been found in the previous experiments, as will be seen by comparing it with the lines given in Fig. 3.

Experiment 5.

In this case a smaller wire was used. It was a piano wire having a diameter of 0.26 mm. After the modulus had been determined at the temperature of the room, the wire was heated to different temperatures by a current around it. The line marked *F* in Fig. 5 gives the curve for the relation of the modulus to the temperature. As in the case of the larger wire it is a straight line, and the inclination is almost identical with that found in a similar way in the preceding experiments. In this experiment, as in several others, the last value was found at the temperature of the room, after the other determinations had been made, in order to see how much effect the heating had produced upon the elasticity. It was found in every case to be a little increased; in this case the increase was about 0.6 per cent. With this wire the smallest weight was 1 kg., and 0.2 kg. was added twice and removed.

Experiment 6.

The wire used in the preceding experiment was heated by a current through it.

In Fig. 5 the points marked with a cross correspond to the results; they are numbered and a line drawn with arrow heads to show the order in which the observations were made. All except number 10 were obtained when a current was flowing through the wire. The temperatures are uncertain, and the results seem so irregular that there is hardly warrant for drawing conclusions from them. A few facts seem to be indicated, however. Comparing 1, 2, and 3 with the last result of the preceding experiment, an increase of elasticity is shown with a weak current, which corresponds with the result found in every case with the larger wire. Comparison of 1 and 10 shows a permanent increase of elasticity from the current through the wire which is about twice as great as the permanent change from heating by the current through the helix.

The wire was so small that the temperature changed easily, and great difficulty was found in keeping it constant; the results finally

obtained are less certain than in the case of the larger wire. At 74° and 92° the results are particularly uncertain, as the wire outside of the glass tube was not sufficiently protected from currents of air. After number 11 the current was again sent through the helix and numbers 12 and 13 obtained, while 14 was without a current. In Fig. 5 the corresponding points are marked with stars. Without much doubt 12 is incorrect. If 13 and 14 are compared, it will be seen that the line *H* between them has an inclination not very different from that of the line *F*; a line from 10 to 13 would be still more nearly parallel to the line *F*.

A comparison of 10 and 14 would indicate that the longitudinal magnetization has decreased the permanent effect of the circular magnetization, though the modulus is still somewhat higher than before the current had been through the wire at all.

EXPERIMENT 7.

As the results for the modulus at the same temperatures obtained with currents around and through the wire were so different, it seemed certain that the magnetization had effect upon the elasticity in one of these cases, if not in both. In order to test the matter it seemed necessary to eliminate the magnetization. For this purpose a double coil was placed on the glass tube, and so arranged that the current would traverse one part of it in one direction and the other in the opposite direction, thus making it non-inductive.

The two points marked with circles near the line *H* in Fig. 5 were obtained with this coil, and the same piece of wire used in the preceding experiments. One determination made at 42° gave a modulus which was considerably higher than any other value with this piece of wire, but no inference can be drawn from a single result which is at variance with the other results. The final result without a current was also higher than had been found previously at the temperature of the room. The results at 72° and 109° compared with the average value found at the temperature of the room show about the same relation of temperature to Young's Modulus as was found with the magnetizing coil. Whether the deviation of the result at 42° is due to an

error of observation or to some other cause I cannot tell. The fact that the other values, and also numbers 13 and 14 found with the magnetizing helix, are higher than those on the curve *F*, is accounted for by the permanent increase of elasticity produced by the current through the wire.

A piece of the larger wire was next adjusted and heated by the non-inductive coil. The line *I* in Fig. 6 corresponds to the results found. As had been found with the magnetizing helix, the elasticity decreases in proportion to the increase of temperature, and the inclination of the line shows that the rate of decrease is very nearly the same.

From these results it seems reasonable to conclude that if longitudinal magnetization has any effect upon Young's Modulus, that effect is very slight.

Experiment 8.

Most of those who have determined Young's Modulus have suspended the wires vertically instead of placing them horizontally as I have done. In order to see if the friction of the pulley introduced an error, pieces of each kind of wire were suspended vertically in a cupboard that had been arranged for a Kater's pendulum. A board was placed in front and openings made through which the same microscopes used in the other determinations could be inserted. Care was taken to have the microscopes horizontal and to so place the weights that the wire would be in the same position with the lighter and heavier weights; some measurements taken without special care with regard to these points were so discordant as to be entirely untrustworthy. Incandescent lamps were used to give illumination for the microscopes. A glass door was between them and the wire, and a screen of paper with openings large enough to permit the illumination of the microscopes. Even with these precautions it was found necessary to turn on the lights for some time before taking observations, in order not to have the variations of temperature introduce an error. The variations during the sets of observations retained were not more than one or two tenths of a degree.

The following results were obtained :—

	Elon.	No. of obs.	L	P	M	M_H	Diff. M_H and M_V.
Large Wire,	1113.95	20	989.08	0.8 Kg.	22631	22950	1.39%
Small Wire,	1316.50	18	990.0	0.4 Kg.	22663	23337	2.88%

M_H is the average result of the determinations made for each kind of wire placed horizontally before they had been heated.

The results obtained in the horizontal position are so concord-

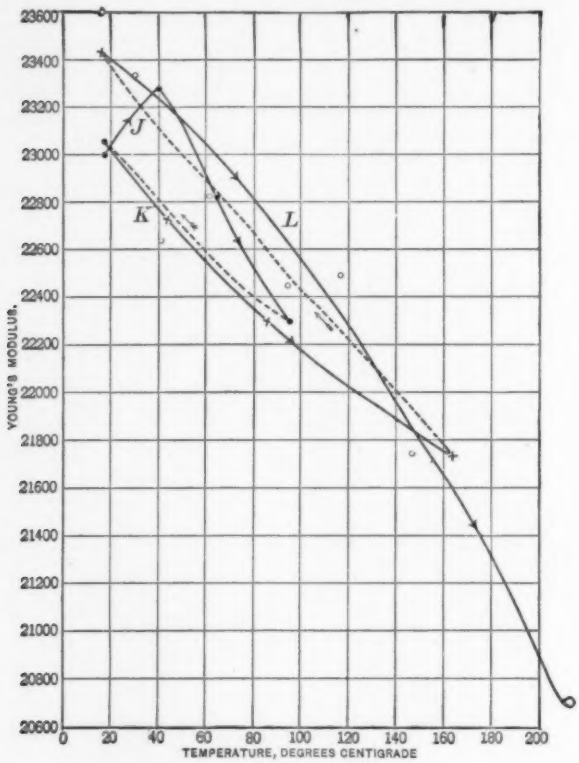


Fig. 6.

ant that it seems quite certain that the effect of the friction of the wheel is always about the same per cent of the result for the same kind of wire, so that the results are as strictly comparable with each other as if obtained in the usual way.

The fact that all of the results obtained are so high is probably because of the kind of wire; the best piano wires manufactured now have a modulus which is considerably higher than those usually given for iron or steel.

Experiment 9.

The same piece of large wire that had been used in the preceding experiment was placed in the tube and its modulus determined when heated by a current through it, the temperature being carefully determined. The following table gives the results obtained, and Fig. 6 shows the graphic representation of the determinations.

DATA FOR CURVES IN FIG. 6.

Number.	T	Elon. for 0.4 kg.	No. of obs.	M
1	20.0	491.02	8	22994
2	18.0	491.29	12	22981
3	18.8	491.19	8	22986
4	23.8	490.36	8	23025
5	40.2	485.06	8	23277
6	65.8	494.75	8	22821
7	95.1	506.69	8	22283
8	18.0	489.75	8	23053
9	43.1	495.75	8	22775
10	84.9	506.05	5	22311
11	145.0	515.36	7	21908
12	163.0	519.57	6	21735
13	17.3	482.25	4	23412
14	29.6	483.69	4	23342
15	40.8	498.50	6	22649
16	63.1	494.38	6	22838
17	95.8	502.70	10	22460
18	117.2	501.72	9	22503
19	152.3	519.94	8	21715
20	218.7	546.27	10	20668
21	17.7	478.41	8	23600

$L = 866.777$ mm. $Q = 0.12566$. $P = 0.4$ Kg.

The first determination was without any current; the second with a current so weak that it changed the temperature only a

fraction of a degree; the third was again without a current. The three results agree within the limits of the errors of observation, showing neither temporary nor permanent effect from so weak a current. The next result, with a little stronger current, shows slight increase of elasticity; while the next, which raised the temperature to about 40°, shows a decided increase. The other two results of this series show a decrease quite similar to that obtained with a current around the wire, while number 8, which was without a current, shows only very slight permanent change from the heating. The next set, including 9 to 13, show regular decrease with increasing temperature, and a considerable permanent change. The last set, including the remaining observations, show a nearly regular decrease, with the exception of numbers 15 and 18; the former is too low and the latter too high to fall on the same line as the others. The permanent change as shown by the final result is considerably greater than before.

The results of this experiment seem to indicate that the effect upon the elasticity due to the *first* application of a current of given strength is different from that produced by a repetition of the current, especially with currents of such strength as to give a temperature in the neighborhood of 40°.

EXPERIMENT 10.

A piece of the smaller wire was used and heated by a current through it. The curve *M* in Fig. 7 represents the results graphically, the arrows indicating the order in which the results were obtained.

The increase of elasticity in the first set comes at a somewhat higher temperature than in the experiments with the larger wire, while a slight decrease is shown at 34°. The other two sets show a decrease of elasticity with increasing temperature, but not a regular decrease. There is a point of inflection in each line showing an increase in the modulus as compared with what would be expected at the temperature, and the point occurs at a higher temperature in the third than in the second set. The possibility of errors of observation is too great to be entirely sure that the positions of the points of inflection are correct; the departure from a straight line is greater, however, than was found in all but a very

few of the experiments made when the wire was heated by a current around it; and the similarity of the two branches increases the probability that they are correct.

Experiment II.

Some of the results obtained for the modulus, when the wire was heated by a current through it, indicated the possibility that the first current sent through the wire had a different effect from subsequent currents. The object of this experiment was to test this subject, and also to see whether previous longitudinal magnetization would modify the effect of the current. Three sets of observations were made with the current always in the same direction, one with the current reversed, and one after the wire had been magnetized longitudinally. Of these five sets of observations the number of observations for the fourth and fifth sets was smaller than had been intended, since unfavorable circumstances made some of the results unreliable, and there was not time to repeat them.

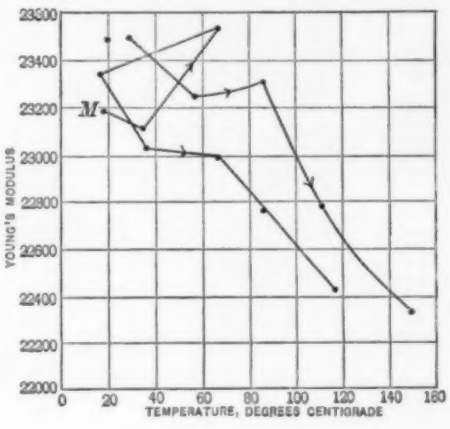


Fig. 7.

The lines in Fig. 8 represent the results. The first and second curves are somewhat similar, only the first does not descend as rapidly at first, and on the return curve shows a higher modulus with a weak current than after the current had ceased and the wire cooled. The first part of the third curve is like the corresponding part of the second; but with decreasing temperature some of the points are higher than at the same temperature on the first part of the curve, and as in the first curve, one is higher than after the wire had cooled.

The few results for the fourth line show no appreciable difference due to reversal of the current.

The fifth line descends less rapidly than the others, indicating that previous longitudinal magnetization may increase the effect that a current has upon elasticity. The results obtained in some

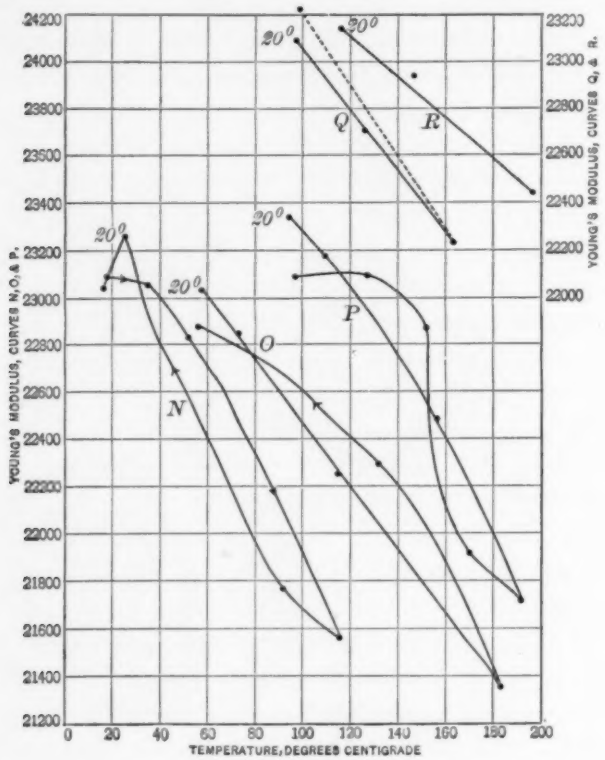


Fig. 8.

of the earlier experiments with wires that had been repeatedly magnetized point toward the same conclusion.

Conclusion.

The results of the experiments show plainly that the elasticity of a piano wire decreases as the temperature increases, with no indication whatever of the maximum that Wertheim supposed to exist between 100° and 200°.

As the decrease has been found to be proportional to the increase of temperature, the relation of Young's Modulus to temperature may be expressed by the formula, $M_t = M_0(1 - at)$.

To determine the value of a, a line was drawn which was the average of the results obtained when the wire was heated by a current through the helix; the modulus at zero degrees was obtained by producing the line. The decrease for 100° for the large wire was 5 per cent, or $a = 0.0005$. Hence the formula is $M_t = M_0(1 - 0.0005\, t)$. For the small wire, $a = 0.00046$.

This result is very nearly the same as that obtained by Kohlrausch and Loomis,[1] but is larger than the value given for a by some other experimenters.

Heating the wire up to about 200° increased the elasticity one or two per cent, the amount varying somewhat in the different experiments.

The various results obtained show that an electric current through a wire does perceptibly modify the elasticity, but that the effect varies with the conditions. The effect of the first current, which is strong enough to raise the temperature to 30° or 40°, was in almost all of the experiments to increase the modulus about one per cent. The heating effects of stronger currents generally decreased it again, but the irregularity of these results compared with the regularity of those obtained when the wire was heated the other way, show that an electric current must modify the elasticity. My results are not numerous or consistent enough to deduce the laws according to which the variations in the effect take place, but indicate that the effect depends to some extent on the previous treatment of the wire and on repetition of the current.

The results obtained with the non-inductive coil show that longitudinal magnetization has no appreciable effect upon elasticity.

This series of experiments was made in the Physical Laboratory of Cornell University, under the direction and with the assistance of Professor Edward L. Nichols.

[1] Ann der Phys. u. Chem. **141**, 480 (1870).

MINOR CONTRIBUTIONS.

ON MAGNETIC POTENTIAL.

BY FREDERICK BEDELL.

Introduction.

IN 1882, Professor Clausius[1] discussed at length the dimensional formulæ of the electrostatic and electromagnetic systems, and derived the dimensions for magnetic pole (m), from a consideration of the equivalence of a magnetic shell and a circuit carrying a current of electricity, as first pointed out by Ampère; thus, —

(1) Area × current = magnetic moment = ml.

In the electromagnetic system, this gives a magnetic pole the dimensions $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$, which accords with Maxwell and subsequent writers. In the electrostatic system, however, this gives to magnetic pole the dimensions $M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-2}$, for which Maxwell gives $M^{\frac{1}{2}}L^{\frac{1}{2}}$. The reasoning of Clausius was apparently correct, and he concluded Maxwell was in error. This conclusion was indorsed by Professor Everett.[2] The discussion[3] which followed "On the Dimensions of a Magnetic Pole in the Electrostatic System of Units" needs merely a mention here.

Professor J. J. Thomson pointed out that the equivalence of a circuit and a magnetic shell, as given by Ampère, is correct only in a medium with permeability equal to unity. The factor μ, denoting the permeability of the medium, should be introduced into the general statement, and (1) should be written —

(2) μ × current × area = magnetic moment.

In the electrostatic system μ has the dimensions $M^0L^{-2}T^2$; hence m has

[1] R. Clausius, "Ueber die verschiedenen Maassysteme zur Messung electrischer und magnetischer Grössen," Verhandl. des naturlust. Vereins des Preuss., Rheinlande und Westfalen, Vol. XXXIX., 1882; also, Philosophical Magazine, June, 1882, p. 381.

[2] J. D. Everett, Philosophical Magazine, May, 1882, p. 376.

[3] Philosophical Magazine, 1882, —

J. J. Thomson:	Vol. XIII., p. 427.	R. Clausius:	Vol. XIV., p. 124.
J. Larmor:	Vol. XIII., p. 429.	J. J. Thomson:	Vol. XIV., p. 225.
J. D. Everett:	Vol. XIII., p. 431.	O. J. Lodge:	Vol. XIV., p. 357.
C. K. Weak:	Vol. XIII., p. 530.	E. B. Sargent:	Vol. XIV., p. 395.

the dimensions $M^{\frac{1}{2}}L^{\frac{1}{2}}$, as given by Maxwell. In the electromagnetic system, permeability is a number without dimensions, and the presence of μ as a factor in (2) does not affect the dimensions of m. Professor Thomson derived these same dimensions from the expression for the work done in carrying a pole around a circuit. He also pointed out that the factor μ should enter into the expression for the attraction between magnetic poles; thus, —

$$f = \frac{mm'}{\mu r^2}. \tag{3}$$

Professor Clausius, however, still considered that he was correct, maintaining that, if a circuit and magnetic shell are equivalent, they must be mutually replaceable, and that a change in the medium would affect one as the other. Professor Everett subsequently concluded that Maxwell was correct, obtaining the dimensions of Maxwell from the galvanometer law and the law of Faraday, although he did not see wherein Clausius was wrong in obtaining his results from the equivalence of a circuit and a magnetic shell. Helmholtz,[1] too, showed lack of confidence in the equivalence of the magnetic shell, and stated that the Ampère-Weber hypothesis must be looked upon as neither verified or even completely worked out. He gave no consideration to the nature of the medium, which, as Professor Thomson pointed out, has a most important bearing on the question. It was left for Dr. Lodge to point out that a distinction should be made between the permeabilities of the medium of the space corresponding to the space of the equivalent shell and of the exterior medium. It was he who pointed out that the magnetic moment of a circuit is equal to the product of the current, the effective area, and the permeability of the *interior* space; the magnetic moment is entirely independent of the permeability of the surrounding space. The failure to recognize this fact was the cause of the erroneous views which had been set forth by such distinguished scientists.

It is the purpose of this paper to consider the effect of permeability upon the magnetic potential due to a magnet or a current of electricity, and to show more definitely the bearing of some of the above relations.

The Dimensions of Magnetic Potential.

The expressions for the magnetic potential at a point, as ordinarily given, are

$$\text{magnetic potential due to a magnet} = V = \Sigma \frac{m}{r}; \tag{4}$$

$$\text{magnetic potential due to a current} = V = I\Omega. \tag{5}$$

[1] Von Helmholtz, Wiedemann's Annalen, 1882, No. 9, p. 42; also, Philosophical Magazine, Vol. XIV., p. 430.

In (5) I denotes the current, and Ω denotes the solid angle subtended by the circuit at the point, the potential of which is under consideration.

In the electromagnetic system these equations lead to the following dimensions for magnetic potentials: —

(6) [from (4)] $$V=\frac{m}{r}=\frac{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}}{L}=M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1};$$

(7) [from (5)] $$V=I\Omega=M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}.$$

These equations (6 and 7) agree with each other and with the dimensions for magnetic potential in the electromagnetic system as usually given, and will be found correct.

In the electrostatic system, equations (4) and (5) give the dimensions:

(8) [from (4)] $$V=\frac{m}{r}=\frac{M^{\frac{1}{2}}L^{\frac{1}{2}}}{L}=M^{\frac{1}{2}}L^{-\frac{1}{2}};$$

(9) [from (5)] $$V=I\Omega=M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}.$$

These equations (8) and (9) are not concordant, although derived from (4) and (5), which gave correct results in the electromagnetic system. Equation (9) will be found to be correct; equations (8) and (4) are not.

If we denote the permeability of the space *exterior* to the magnet by μ_e, instead of (4), we should have

(10) $$V=\Sigma\frac{m}{\mu_e r},$$

which accords with (3), and is true in any system of units.

In the electromagnetic system the dimensions of μ are $M^0L^0T^0$; hence μ does not affect the dimensions in this system, and (6) and (7) are correct.

In the electrostatic system the dimensions of μ are $L^{-2}T^2$. Hence the dimensions of magnetic potential derived from (10) are: —

(11) $$V=\frac{m}{\mu_e r}=\frac{M^{\frac{1}{2}}L^{\frac{1}{2}}}{L^{-2}T^2L}=M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2},$$

which agrees with (9), and is correct.

In a Uniform Medium the Magnetic Potential due to a Current is Independent of Permeability.

The introduction of μ in the expression for the magnetic potential due to a magnet (4) gives the same dimensions to magnetic potential (see 11), as derived from (5), the expression for the magnetic potential due to a current.

That the permeability of the medium should be taken into consideration, as above, in the expression for the magnetic potential due to a magnet is clear; but should we not expect the permeability to enter in a similar manner in the corresponding expression for the magnetic potential due to a current (see 5)? The equivalence of a current and a magnetic shell as stated by Ampère would lead to this conclusion. Such, however, is not the case; equation (5) holds true as it is written. The magnetic potential at a point due to a current is independent of the permeability, — a fact the explanation of which merits attention.

In *air*, the magnetic potential at a point due to a current may be obtained from the principle of the magnetic shell as follows. Consider the circuit replaced by a shell consisting of elementary magnets, with axes in the direction of the thickness of shell. If r and r' are distances of the point P from the two poles of one of these elementary magnets, of length l and pole strength dm, the magnetic potential at the point P, due to this magnet, is

$$dV = \frac{dm}{r} - \frac{dm}{r'}. \tag{12}$$

Inasmuch as the length of the magnet is small compared with its distance from the point P, for r' we may write $r + l\cos\alpha$, where α is the angle between the axis of the magnet and a line connecting it with the point P. Hence

$$dV = dm\left(\frac{1}{r} - \frac{1}{r + l\cos\alpha}\right) = \frac{ldm\cos\alpha}{r^2}, \tag{13}$$

the last term being obtained by approximation. But from the equivalence of a magnetic shell in air (see 1) we have

$$ldm = IdA; \tag{14}$$

where dA represent the polar area of an elementary magnet. Making this substitution in (13), and remembering that $dA\cos\alpha \div r^2 = d\Omega$, the solid angle at P subtended by the polar surface of the elementary magnet, we have

$$dV = Id\Omega; \text{ and } V = I\Omega. \tag{15}$$

This reasoning holds true for air; but (12), (13), (14) are true only for air, or a medium whose permeability is unity.

For *media other than air* equation (15) still holds. If μ_e is the permeability of the space *exterior* to the space occupied by the magnetic shell to which the current is considered equivalent, instead of (12) and (13), we have [see equation (10)]: —

$$dV = \frac{dm}{\mu_e r} - \frac{dm}{\mu_e r'} = \frac{1}{\mu_e}\frac{ldm\cos\alpha}{r^2}. \tag{16}$$

If μ_i is the permeability of the *interior* space, or that occupied by the equivalent magnetic shell, instead of (14), we have

$$ldm = \mu_i I dA. \tag{17}$$

Substituting (17) in (16), we have

$$dV = \frac{\mu_i}{\mu_e} \frac{I dA \cos \alpha}{r^2} = \frac{\mu_i}{\mu_e} I d\Omega. \tag{18}$$

$$V = \frac{\mu_i}{\mu_e} I\Omega. \tag{19}$$

This is the expression for the magnetic potential at a point P, due to a current I, in a circuit subtending the angle Ω at the point P, when μ_i is the permeability of the space occupied by an hypothetical equivalent shell, and μ_e is the permeability of the surrounding medium.

In all cases usually considered $\mu_i = \mu_e$; that is, in any uniform medium, the magnetic potential due to a current is

$$V = I\Omega, \tag{20}$$

and is independent of the permeability of the medium.

A METHOD FOR THE STUDY OF TRANSMISSION SPECTRA IN THE ULTRA-VIOLET.

BY ERNEST NICHOLS.

THE method to be described is a very simple one, and has long been in use in other branches of spectrum work. It is to photograph, side by side, two spectra arising from different portions of the same slit; one a spectrum of sunlight, the other a spectrum of sunlight changed by passing through a layer of the substance to be examined. The apparatus used in the present study consisted of a Rowland concave grating with a 2 m. radius of curvature, and 568 rulings to the millimeter; a narrow slit 4 cm. long; a heliostat with a silver mirror, and a dark box or camera. The grating was placed in the path of a beam of sunlight sent through the slit in the window shutter from the heliostat; and so adjusted that the image of the slit, due to the grating mirror, was reflected back upon the slit, and was of the same size as the slit itself. According to the approximate law for the grating, the spectrum should come to focus on the circumference described on the line connecting the center of the grating and the middle of the slit as a diameter. The film holder in the camera was warped to form an arc of this circle. The camera was further provided with a sliding front. After the camera had been adjusted so that the

whole visible spectrum was in focus on a part of the film holder, a flexible celluloid film, 20 × 5 cm. was slid into the holder where the extreme violet and ultra-violet rays would fall upon it during the exposure. To give a negative of anything like equal intensity throughout its length a differential exposure was required. The film was exposed by sliding back the front of the camera, exposing first the extreme ultra-violet and in succession the longer wave-lengths, until a region near the G lines was reached; the motion was then reversed and the opening gradually closed. An exposure of from 30 to 40 seconds for the shortest wave-lengths, and from 2 to 4 seconds for the region of the G lines, was found, under the conditions of the present study, to give the best results. But little store can be set by such values, for the requisite exposure varies inversely as the width of the slit, the reflecting power of the heliostat mirror and grating, and the rapidity of the films; and varies directly as the dispersion. The ratio of the exposure required by the two extremes of this region under ordinary conditions should, however, be about as 16 to 1. Plates of the transparent solids to be examined were brought up against the back of the slit in such a way as to cover the lower half of it. In this way two spectra were brought together, side by side, in the camera. The spectrum from the upper half of the slit was the record of unchanged sunlight, and that from the lower half, of sunlight which had passed through the substance under examination. Solutions were examined in a glass cell equal in depth to the length of the slit, and filled half full of the solution to be studied. The only difference between the two spectra thus obtained in the camera, must be due to the characteristic absorption of the solution in question. The estimation of wave-length in the ultra-violet spectrum, which might otherwise require more or less complicated machinery, is very easily and accurately made by reference to the Fraunhofer lines. In the present study, wave-lengths were written down in accordance with the photographs of the solar spectrum made by Professor Rowland in 1888. The method gives quantitative results of only the roughest sort, for quantitative measurements must depend upon the estimation of depth of shade, which it is always difficult to determine with exactness; and behind this lies the uncertain relation between the change produced in silver salts and the amount of light received by them. Fortunately, in work of this sort qualitative results are the more important, and quantitative results, on account of their dependence on the thickness of the layer, or upon the degree of concentration of the solution under examination, are only secondary. The present paper contains the results from the examination of thirty-eight different substances, twelve of which showed more or less selective absorption in the region studied. The results appear in Tables I. and II. Among the substances showing selective absorption were several kinds

of glass, a sample of white mica, and solutions of uranium nitrate and uranium acetate, and a fluorescent solution obtained in the charring of sucrose with strong sulphuric acid.

Reproductions of the photographs of a number of these spectra are given in the accompanying plate.

TABLE I.

SOLUTION.

(Thickness of layer, 1 cm.)

Substance.	Concentration.	Region studied.		Transmission.
		λ=(in μ)	to λ=	
Water (distilled)	—	0.41	0.31	Absorption very slight and not selective.
Alcohol (absolute)	—	—	—	Same as Water.
Lithium chloride	2 grams in 20 cc. water	—	—	Same as Water.
Potassium chloride	2 grams in 20 cc. water	—	—	Same as Water.
Sodium chloride	2 grams in 20 cc. water	—	—	Same as Water.
Strontium chloride	2 grams in 20 cc. water	—	—	Same as Water.
Thallium chloride	1 gram in 20 cc. water	—	—	Same as Water.
Indium chloride	½ gram in 20 cc. water	—	—	Same as Water.
$(NH_4)_2Al_2(SO_4)_4$	Saturated at 0°	—	—	Same as Water.
$K_2Al_2(SO_4)_4$	Saturated at 0°	—	—	Same as Water.
$(NH_4)_2Fe_2(SO_4)_4$	Saturated at 0°	—	—	See Fig. 1.
Iodine in CS_2	½ gram in 20 cc. CS_2	0.43	0.325	Total from .43 to .386; decreases to .377; None from .377 to .325.
Chlorophyll in alcohol	—	0.4	0.325	None.
Fluorescent substance[1]	—	0.45	0.30	Absorption slight up to $\lambda = 0.40$. Then gradually increases as λ becomes less.
Uranine	Weak solution in water	0.4	0.325	Nearly total, absorption uniform.
Uranine	Stronger solution in water	0.4	0.325	Absorption considerable, but uniform.
Fluorescein	—	0.4	0.325	Practically same as Uranine.
Stramonium seeds	Alcoholic solution	0.4	0.325	None.
Uranium nitrate	4 grams in 20 cc. water (saturated)	—	—	See Fig. 2.
Uranium acetate	20 cc. water 20 drops HCl	—	—	See Fig. 3.

[1] Obtained by charring sucrose with strong sulphuric acid.

TABLE II.

SOLIDS.

Substance.	Thickness (in mms).	Region studied.		Transmission.
		$\lambda=\mu$ to	$\lambda=\mu$.	
Rock salt	6.0	0.4	0.31	Total.
Glass (common window)	2.4	—	—	Absorption complete from $\lambda=0.325$ on.
Mica (white)	0.07	—	—	Absorption begins at $\lambda=0.33$ and becomes complete at $\lambda=0.30$.
Quartz	3.2	0.4	0.30	Total.
Gypsum	3.6	0.4	0.30	Total.
Alum (solid)	3.2	0.4	0.30	Total.
Fluorspar	3.5	0.4	0.30	Total.
Plate glass	8.0	0.4	0.32	Like common glass above.
Flint glass	3.5	0.4	0.31	Like common glass above.
Black glass	2.0	0.4	0.30	None.
Red glass	2.0	0.4	0.30	None.
Brown agate	3.1	0.4	0.30	Uniform, nearly opaque.
Uranium glass	31.0	0.4	0.30	None.
Tourmaline (blue)	5.9	0.4	0.31	None.
Tourmaline (green)	5.0	0.4	0.31	None.
Cobalt glass	2.0	—	—	See Fig. 4.
Dydinium glass	7.5	—	—	See Fig. 5.
Calcite[1]	10.0	0.4	0.31	Total.

[1] Plate cut perpendicular to optic axis.

COLGATE UNIVERSITY, April and May, 1894.

THE PHOTOGRAPHY OF MANOMETRIC FLAMES.

BY WILLIAM HALLOCK.

WHILE occupied with a research upon the use of the nasal cavity in singing, which had been suggested by Dr. Floyd S. Muckey, and was largely carried out by him, it became desirable to record our results by photographing the manometric flames which we used.

For the attempted analysis of the voice, with and without the use of the nasal cavity, we arranged a set of König resonators, each having a mano-

metric capsule in the back of it, connected by a rubber tube to the series of small gas burners, situated one above the other. The resonators were tuned to Ut_2 and its first seven over-tones, Ut_3, Sol_3, Ut_4, Mi_4, Sol_4, "number 7," and Ut_5. The whole arrangement was identical with the familiar apparatus of König.

The flames being viewed in a rotating mirror appeared as a bright line when that particular resonator was not responding, and as a wavy line when the air in the resonator vibrated and caused the membrane of the manometric capsule to vibrate also, and the flame to jump up and down.

It is impossible to photograph what one sees in the rotating mirror on account of the irregularities of the mirror and its rotation, hence some other method of recording the vibration of the flame must be devised. Professor Merritt[1] has accomplished this by causing the sensitive plate to travel across the back of the camera, so that at each vibration the image of the flame would fall upon a new part of the plate, and thus be continuously recorded. It might be possible to cause the lens to move across the front of the camera, and thus accomplish the same result.

In our case a camera was mounted upon a piece of board, which was pivoted approximately under the optical center of the lens, thus enabling us to rotate the camera about the center of the lens. When the camera is so rotated, the image of any object in front of it will traverse the plate, and be photographed as a line, or otherwise, as the case may be. A large black screen was so arranged on the stationary part of the apparatus that the flames, and the flames only, were visible to the camera through a slit about an inch wide in the screen, directly in front of the camera in its central position.

The method of operating was as follows. The camera was first focused upon the flames. The cap may be left off all the time, and no stop is needed. The plate-holder was then placed in the camera, which was directed toward the black screen on one side of the slit. When all was ready, and the flames were behaving properly (as seen in the rotating mirror), the slide was withdrawn from the plate-holder and the camera quickly rotated through about 90 degrees, until it pointed toward the screen on the opposite side of the slit; the slide was then replaced in the holder, and the plate was ready for development. The eight flames of our apparatus recorded themselves either as straight lines or as wavy lines, according as their resonators responded to overtones or not. The record was made while the camera was pointed toward the flames through the slit in the screen. Springs were arranged to rotate the camera at definite and constant velocities, but it was found that it could be rotated with the hand quite well enough. When a poor lens (not rectilinear) is used, or great accuracy is

[1] Ernest Merritt, PHYSICAL REVIEW, Vol. I., p. 166, 1893.

necessary in the comparison of the number of vibrations in any two notes, two momentary exposures of the plate to the flames, with the camera at rest, and so stationed that the images fall once on one edge of the plate and again on the opposite edge, will give the fiducial points from which to count, eliminating any distortion due to the lens. By this means very common lenses may be used, with very large apertures.

Heretofore it has been considered necessary to increase the actinic power of the little flames by surrounding them with oxygen, or other means. We found this unnecessary if we took the precaution to turn up the flames until a characteristic bright point appeared just at the top of the blue. We used an old 4×5 camera with a fair 5×7 lens, and Cramer's "isochromatic C." dry plates with ordinary developer. A good negative was obtained in this way with a lens costing only fifty cents. This lens was plano-convex, with a diameter of three inches and an eight-inch focus, and was used with full aperture.

PHYSICAL LABORATORY, COLUMBIA COLLEGE,
NEW YORK, August, 1894.

NEW BOOKS.

A Treatise on Astronomical Spectroscopy: being a translation of Die Spectralanalyse der Gestirne. By Professor Dr. J. SCHEINER. Translated, revised, and enlarged by EDWIN BRANT FROST. 8vo, pp. 482. Boston, Ginn & Co., 1894.

When the German original of this work appeared in 1890, the thought must have occurred to many that rarely had any book come so perfectly in season. A new science had recently arisen by differentiation from astronomy and physics. An immense quantity of observations, of all degrees of value, had been made, and then recorded in journals of every character. Theories, often of most contradictory natures, had been published. What was needed was a man competent and willing to reduce the observations and to discuss the theories. Scheiner tried to do this and more; and his book has been of untold usefulness to the scientific world. In the past four years, however, material has been accumulating at an enormous rate, especially in this country; and the time has come for a revision of Scheiner's treatise. It is most gratifying to American scholarship to realize that this work was undertaken by a professor of Dartmouth College, Mr. Edwin B. Frost, and that in his revision the amount of new matter introduced entitles the book to be called an enlargement as well.

Professor Frost's work is, in the highest degree, satisfactory. His arrangement of the subjects is the same as Scheiner's. The book is divided into four parts. Part I. treats of spectroscopic apparatus. Part II. gives a discussion of Kirchhoff's Law and Doppler's Principle, as these subjects were understood in 1890. Part III. deals with the results of spectroscopic observations of the sun, the planets, comets, nebulæ, the stars, and the aurora. There is also a chapter on the displacements of spectral lines. Part IV. contains various spectroscopic tables, notably Rowland's new table of standard wave-lengths and Kayser and Runge's lists of iron lines. The bibliography, one of the most important features of Scheiner's original work, is extended so as to include all publications through the year 1893. The changes and additions in the revised work are in every case distinct improvements.

By far the most important of the changes is the substitution of Row-

land's scale of wave-lengths for the Potsdam scale, which is based upon Müller and Kempf's work. Apart from the question of relative accuracy of these two scales (and a good argument might be based on this), the great advantage arising from the use of Rowland's scale is the securing of uniformity over almost the entire scientific world. With Young, Keeler, Pickering, and Campbell in America, and Kayser and Runge in Germany using Rowland's scale, it would be folly for any observer to adhere to a different system. Uniformity is of the utmost importance; and Professor Frost has rendered science great service in reducing to Rowland's scale the lists of measurements, which for years to come will be standards for reference.

The new observations and theories which have been published in the past four years receive due attention. The book is wonderfully well up to date. It is true that since its publication fresh light has been thrown on several disputed points, especially on the spectrum of the Great Nebula in Orion and the theory of Nova Aurigæ; but in stellar spectroscopy only a weekly or monthly magazine can hope to lay any claim to having stated all the facts known. Professor Frost has profited, too, by the kindly assistance of other American investigators. Campbell contributes a section on the reduction of spectroscopic observations of motions in the line of sight. Young publishes a partial revision of the chromospheric lines. Hale's spectroheliograph is fully described.

Altogether, Professor Frost's book is a great step in advance, as was made necessary by the unexampled progress of astrophysics itself. But, in spite of this, there are certain points, common to the original and the newly published books, which cannot be regarded but as defects; and again there are many statements which should be received with caution. It is in the first part of both books, that devoted to spectroscopic apparatus, that they are least satisfactory. It is true that the descriptions of the apparatus in use in the great observatories is most complete; many interesting theories and discussions are given; and experiments such as those on the effect of changes in temperature upon the index of refraction are reported in full. In each one of these particulars, too, the American book is superior to the German. The point where they both fail is just here: the ordinary observer who has a telescope or spectroscope or both, and who wishes to do what he can under existing conditions, gets very little help or suggestion. From a German standpoint this is not very important; but here in America, where there are innumerable colleges and also amateur observers, each with instruments, often extremely good, and each anxious to take part in this new and fascinating work in astrophysics, it is the most serious matter possible. No one can tell how much work, and good work too, might be done if only proper guidance could be given. It would have been much better if, instead of describing so many finished

forms of apparatus, more space had been given to the provisional forms, or to the home-made pieces that every observatory has to use. In any case, attention should surely have been called to Keeler's paper in the *Sidereal Messenger*, X., 1891, "On Elementary Principles governing the Efficiency of Spectroscopes for Astronomical Purposes." Few articles so helpful as this have ever appeared. This same section of the book could have been greatly improved, too, if the instrumental adjustments and corrections had been described more in detail. The author and the translator, as Professor Frost modestly calls himself, may have left these out purposely; but surely the book would have had a much wider field of usefulness if they had been included.

There are other points of weakness, too. One is, of course, the system of classification of the stellar spectra; and to the defects of this Professor Frost himself calls attention, saying that many spectroscopists regard any classification as provisional in the light of our present ignorance. This last fact is especially true for two reasons, one being that in Scheiner's classification undue importance is attached to the visual spectra; the other being that the interpretation of the observed spectra is so exceedingly uncertain. In our complete ignorance as to the cause of so many stellar radiations, and in the almost complete absence of experimental data as to changes produced in spectra by alterations in temperature or pressure, or by the presence of other substances, it certainly seems unwise to make use of theoretical explanations. The discontinuous spectra emitted by the stars may be due to temperature effects, they may be caused by electrical discharges, or at least accompanied by chemical changes; and, as we know so little about it, the application of theoretical deductions from Kirchhoff's law is decidedly unjustifiable. The question, too, of identification of the stellar lines with those due to substances known to us is by no means so easy or so certain as Scheiner would have us think. Identification of lines is one of the most arduous tasks of the spectroscopist; and it is by no means a matter of accuracy of measurement simply. It is difficult enough in many cases in the solar spectrum; but it is many times more so with stellar spectra, owing to physical changes in the spectra, such as the disappearance of lines and groups, changes in relative intensities, new shadings on different sides. Of course it is very easy to account for these changes on theoretical grounds unsupported by experiment, especially if the theory is assumed; but the reader of the book now under discussion receives, I think, an entirely wrong impression of our actual *knowledge* of these subjects.

The book is, however, a perfect storehouse of facts; and through it the work of every investigator will be made easier. One wishes that more attention had been called to what we do not know, so that the future work

of new observers could be directed into useful courses. But the book, as it is, is one that will be instantly read by all workers with telescope and spectroscope, and will be kept at hand for daily reference. The appearance of the book is a credit not alone to the author, but also to the publishers, who surely show their deep interest in scientific education in its production. The paper, type, diagrams, and illustrations are all that could be desired.

JOSEPH S. AMES.

Magnetische Kreise, deren Theorie und Anwendung. By Dr. H. DU BOIS. 8vo, pp. xiv+382. Berlin, Julius Springer, 1894.

To one who has read the titles of the numerous books on magnetism and its applications which have appeared during the last few years, it might seem that to produce anything new in this field was an impossibility. The volume before us demonstrates, however, the falsity of such a conclusion. Although Dr. du Bois's book contains little matter which is new in the sense of appearing for the first time in print, yet the choice and arrangement of the material, and especially the manner in which it is presented, render the book quite different from works on magnetism which have heretofore appeared. It cannot be likened to Ewing's treatise, for the effects of magnetism upon the physical properties of matter, and the differences exhibited by various bodies in their behavior when subjected to magnetizing forces, are not discussed; while methods of magnetic measurement, although mentioned, are treated only briefly. Still less is it to be compared with the works of S. P. Thompson, for the methods of presentation are entirely different. And finally, although the more important practical applications are briefly described, yet the book has little in common with most technical works on magnetism.

Speaking generally, it may be said that the aim of the book is to present together, in a connected and consistent form, *all* the more important methods of treating magnetic problems. The chapters dealing with technical applications serve to illustrate the methods and principles discussed. In consequence of the magnitude of the task which the author has undertaken, the treatment is in many cases of necessity brief. Yet the reader will be able to obtain here a broad view of the subject, such as could otherwise be reached only by a long and tedious search through the original literature.

The book is divided, naturally, into two parts, the first of which concerns itself wholly with magnetic theory, while the second deals with practical applications, and methods of magnetic measurement. A general knowledge

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of the subject on the part of the reader is assumed; yet the first two chapters are devoted to a review in elementary form of fundamental phenomena and notions, and will be found interesting reading on account of several novel features in the mode of treatment. Evidently impressed by the ridicule that has recently been thrown on magnetic poles and the law of inverse squares by a well-known English writer, the author has avoided the use of the term "pole" altogether. In the first chapter the innovation is very successful. Strength of field is defined, for instance, by reference to the current induced in a small test coil, when the latter is rotated through 180°. In the second and following chapters, however, it becomes difficult to dispense with the conception of pole-faces, and the terms "end" and "end element" are introduced. The new terms are used in the same sense in which the expressions "pole" and "pole-face" are rightfully employed, and the advantage gained is not obvious. Yet it must be admitted that the new expressions soon lose their strangeness, and are by no means inappropriate. The distinction made in the first chapter between paramagnetic and ferromagnetic bodies — viz. that the latter exhibit hysteresis, while the former do not — seems uncalled for.

After two chapters devoted to the essential features of the ordinary mathematical theory of permanent magnets, and of induction in isotropic media, we come, in Chapter V., to the problem of the induction in a toroid, or anchor ring, containing a radial slit. The author may be said to base his whole treatment of discontinuous magnetic circuits upon this typical case. The discussion in Chapter V. is in accordance with the theory developed in the preceding chapters, and is quite different from that with which, I think, most American readers are familiar. It will perhaps prove of especial interest for this very reason. Successive approximate expressions are developed for the demagnetizing factor and the leakage coefficient, and are shown to agree with the results of experiment within reasonable limits.

Part II., which comprises about two-thirds of the book, begins with a discussion of circuits in which the magnetizing coils are unsymmetrically distributed. The tendency of the total flux of induction to remain the same in all parts of such circuits, in spite of the lack of symmetry in the original magnetizing field, is explained, not by reference to the analogy of electric currents, but as a result of the reaction of the "end elements" at points of the circuit where leakage occurs. Strangely enough the explanation of this fundamental property of magnetic circuits is rarely given, except perhaps by a fleeting reference to the electric analogy, which must often appear to the student as not wholly satisfactory. The two clearly written paragraphs which Dr. du Bois devotes to the "compensating action of leakage" are therefore doubly welcome. The discussion

of Hopkinson's synthetic method, which follows, does not do justice to this most important method of treating magnetic problems. Here, as in many other parts of the book, English and American readers will regret the absence of concrete *numerical* illustrations.

Chapter VII. is devoted to the analogies between magnetic induction and various forms of current flow. A brief history of the development of such analogies, and a table in which they are compared, add to the interest of the chapter. The greater portion of Chapter VII. naturally deals with the electric current analogy, and Dr. du Bois points out quite clearly many of the dangers which must be avoided in its application. That such dangers exist is only too true. I have known a serious attempt to be made, for example, to compute hysteresis loss by reference to a formula analogous to the well-known C^2Rt of Joule. But any analogy may lead to equally ridiculous results when carelessly applied. A perfect analogy would be an identity. The statement, or rather the implication, made in Chapter VII. that the analogue of Ohm's law may lead to false results, even when applied with due caution, does not, however, appear to be justified by the example cited. The author lays great stress on results obtained with a uniformly wound anchor ring containing a narrow slit. Experiment shows that the leakage coefficient, defined as the ratio of the total flux to that portion which goes directly across the slit from pole to pole, grows less as the iron becomes more saturated, and apparently approaches unity as a limit. A superficial consideration of the problem from the standpoint of the electric analogy would indeed lead one to expect an opposite result. But if we imagine the iron ring replaced by a conductor immersed in a poorly conducting liquid, provide a *distributed* E. M. F. to correspond with the uniform winding of the magnetizing coil, and imagine the specific resistance of the conductor to approach that of the liquid, the conclusions would correspond exactly to the results obtained by the experiments referred to.

In Chapters VIII. and IX. are discussed the various forms of magnetic circuit as they occur in the dynamo, electro-magnet, and the transformer. The limited space at the author's disposal has prevented a detailed treatment of any of the cases considered, so that the designer will find little here which is new. But for the physicist who wishes to obtain a glimpse of the applications of his science, these chapters will prove most interesting reading. Of especial interest is the description of a large electro-magnet, constructed by the author for developing strong fields. The essential features to be considered in the design of such magnets are discussed in some detail, and the result obtained, viz. a field of $H = 40{,}000$, shows that the principles explained have been successfully applied.

The book closes with a brief but comprehensive résumé of the various

methods of measuring H and B. The discussion of the magneto-optical method for the measurements of strong fields is deserving of special mention.

ERNEST MERRITT.

"Light": An Elementary Text-Book Theoretical and Practical for Colleges and Schools. By R. T. GLAZEBROOK, F.R.S., Assistant Director of the Cavendish Laboratory. Cambridge Natural Science Manuals. 8vo, pp. x + 213. University Press, 1894.

This little volume of 200 pages is an elementary text-book, and represents a course for medical students given at the Cavendish Laboratory. It is designed to be used by beginners who are accompanying their class work with experiments. In connection with the text about thirty simple experiments are described as practical exercises for the pupil, illustrating the principles and laws studied. The first six chapters deal with the laws of light intensity, the measurement of its velocity, giving brief outlines of Fizeau's and Foucault's, as well as of the Astronomical methods, reflection at plane and curved surfaces, and the properties of lenses. In the three concluding chapters are discussed the eye in its relation to vision, the more important optical instruments, the leading facts relating to color and color blindness, the formation of the spectrum, and the difference in character between the spectra of solids or liquids and gases.

Many questions and examples are given at the end of each chapter.

That Light is a wave motion in the ether is mentioned, but the whole treatment is by the method of geometrical optics. It is not even attempted to deduce the law of refraction from the wave theory. This entire exclusion of physical optics from an elementary text-book seems to us a mistake. Would it not be more profitable for the student to spend less time over calculations relating to lenses and curved mirrors, and learn something of interference, diffraction, and the colors of thin films, and so in a way be able to appreciate the grounds of the wave theory, and to think of light as a wave motion? Probably the pupil would sacrifice some facility in solving lens problems, but he would have a far deeper insight into the science of optics.

The diagrams are excellent, and we are glad to note the tracing of *pencils* of light instead of *rays* in optical instruments, as this important point is too often neglected.

A. L. KIMBALL.

The Steam Engine and Other Heat Engines. By J. A. EWING, F.R.S., Professor of Mechanism and Applied Mechanics in the University of Cambridge. 8vo, pp. xiv + 400. Cambridge Press, 1894.

This is an outgrowth of the article prepared by Professor Ewing for the Encyclopedia Britannica, and is expanded into a treatise intended to serve as a university text-book. It covers, therefore, a field a little different from that of Cotterill and Peabody on the thermodynamics of the steam engine, and different from that of Thurston and Whitham, in the field of constructive steam engineering. It partakes a little of both, and in addition devotes some most interesting space to air, gas, and oil motors.

From his nearness to original sources, the author has been able to present in his first chapter some most excellent illustrations of the early steps in the utilization of heat for steam making, and then devotes 130 pages to a discussion of the theory of the steam engine as a heat motor. This discussion seems to be most satisfactory and useful; without taking the position of a full presentation of the theorems of thermodynamics, it is in no way open to the criticism of being superficial. The discussion of the Carnot cycle and its realization in the actual engine is especially clear. At the close of this discussion some space is devoted to the reversal of the heat cycle, in refrigerating machinery, and likewise to the use of engines with more than one working substance. The chapters on the actual behavior of steam in the cylinder and the effects of superheating and jacketing, high speed, etc., are particularly full.

The second part of the work covers the development of the compound and multiple-expansion engines, the testing of engines, the kinematics of valve gear and governing, and the forms of boiler and engine. The references under these latter heads to a number of modern forms, and the results of tests of steam turbines, give special interest to this part of the discussion. The concluding chapter on air, gas, and oil engines, is of particular interest in America as the development of the direct combustion engine has been much wider in England and on the Continent than here. Within the limits which the author set himself the discussion is full and interesting, but of course goes no further than a presentation of typical arrangements. The book concludes with a table of the properties of saturated steam from a temperature of 32° to 401° Fahrenheit, the superior limit corresponding to a pressure of 250 pounds to the square inch.

The book is of a type excellently well adapted to serve as a text-book for students in engineering where the design, construction, and operation of the steam engine is not to be made an exclusive specialty. It will serve also most admirably as a text-book and work of reference for this latter class where lectures and other exercises are to supplement the course laid

down in the text-book which is used. It will form a valuable addition to the library of every engineer.

F. R. HUTTON.

Elementary Hydrostatics. By JOHN GREAVES, M.A., Fellow of Christ's College. 8vo, pp. xi.+204. Cambridge, University Press, 1894.

This unassuming title carries with it a wholesome lesson to contributors to the scientific literature of our day. Science can scarcely gain an advance through the multiplication of text-books of a general nature, merely stating the old principles in new garb and covering fields already fully harvested. Because there is an occasional John Tyndal who may translate the deeper truths of science into terms of the popular tongue, the needs of modern science are none the less sharply confined, in general, to progress along specific lines. Every new book which does not aim to fill a distinct want in the literature of the subject treated, lays an unmerited burden upon the workers in that field, who must individually satisfy themselves of its purport.

The volume under discussion is designed for students preparing for the First Part of the Mathematical Tripos, and in the fulfillment of this purpose has its birthright. This may serve as an opportune occasion for the writer to register, in no feeble terms, his entire disbelief in the scientific value of books written for such a purpose, and his lack of sympathy with the English idea of making every permanent feature give way to temporary triumph in these examinations. The treatment of Hydrostatics is essentially from the abstract side, and, as such, could hardly be expected to be the most useful to the student who wishes to grasp the subject in its concrete bearings. There are evidences of good proof-reading and of excellent printer's work. The lavish use of broad-faced type, to make prominent the salient points, is, æsthetically, a mistake, and its object might have been accomplished more tastily.

The definitions, usually, are good. However, in the attempt to exclude everything but the idea defined, the author has laid himself open to the criticism that some of his definitions are not broad and full, as vehicles of the general idea, but are, possibly, too special and technical. In Chapter I. the relations between stress and strain and the definitions of the properties of matter are clear, concise, and well illustrated by examples. On page 11, a possible weakness in treatment is shown in the dependence of the proof upon § 16 which comes later on in the book.

In general, the whole discussion might be made clearer by the judicious use of more diagrams and by the graphic representation of the unit areas and volumes with the forces acting upon them. The attempt is made to

show everything on one plane where isometric projection would add largely to clearness. A potent lesson in this regard may be drawn from the treatment, by Professor Church, of the subject of Hydrostatics in his *Mechanics of Fluids*, pp. 515–603. As a text-book for preparation for the Mathematical Tripos, the volume seems quite satisfactory. As a broad treatment for general purposes, it is subject to some criticism.

ELON HUNTINGTON HOOKER.

Recent Text-Books in Laboratory Physics.

The little manual[1] by W. G. Woollcombe is evidently intended to precede the author's *Practical Work in Heat* published last year, and is presumably to be followed by others covering the remaining divisions of physics. It is an excellent introduction to laboratory work, covering fifty experiments. Of these, nineteen are devoted to measurements of length, including areas and volumes. The next twenty cover measurements of mass, and the remaining eleven are divided between barometers, the simple pendulum, and capillarity.

The book is thoroughly practical, the experiments well selected and arranged, and the instructions and explanations sufficiently full in detail to render an ordinarily intelligent boy of fourteen independent of the teacher.

All the experiments are quantitative, and as many of them involve the use of liquids, the student receives a good drill in manipulations. It would seem desirable to conduct a course of this sort in a room fitted up for chemical experiments, with convenient sinks, troughs for mercury, and possibly aprons for the students.

The author's rule (on p. 13) to express all lengths in centimeters, instead of the customary use of millimeters for small dimensions, if universally adopted, would do much to diminish mistakes and confusion.

In the laboratory manual[2] by H. N. Chute, the subject is covered in some 128 experiments, of which general measurements and mechanics take forty-three, heat seventeen, magnetism and electricity twenty-five, sound eleven, and light thirty-two. Half a day each week is supposed to be given to the laboratory, and in addition home work is required in the preparation of reports on the results.

The book will commend itself to those teachers who wish to utilize their lecture apparatus in the laboratory, as a large number of the experiments are qualitative, demonstrating natural laws and repeating those of the

[1] *Practical Work in General Physics.* By W. G. WOOLLCOMBE. pp. xii+83. The Clarendon Press, 1894.

[2] *A Physical Laboratory Manual for Use in Schools and Colleges.* By H. N. CHUTE. 8vo, pp. xiv+213. D. C. Heath & Co., 1894.

lecturer. Some of the practical hints are excellent, and the shelf of reference books in the laboratory, if the students can be encouraged to use them in the way proposed, should be an important addition.

The volume is well printed and illustrated, and the binding of a nature to stand the rough usage of a laboratory.

In *Practical Lessons in Physical Measurement*,[1] by Alfred Earl, the hope is expressed by the author that the treatise may serve as a Practical Arithmetic and Practical Grammar, as well as an Introduction to Practical Physics. He might have added Practical Geometry and Practical English Composition, on the strength of Chapters V. and VIII.

The aim of the book is to take young boys and give them a thorough foundation for future experimental work. The course begins in the very rudiments of measurement, and the selection and construction of standards is thoroughly discussed. The whole 270 experiments are devoted to measurements of length, mass, and time, and every detail tending to accuracy is carefully considered.

Great stress is laid by the author on accuracy of expression and correctness of language in the preparation of the reports on the results. The use of the pen in the description of the experiment is considered of equal importance to the use of the instruments themselves.

The book would be much improved by a chapter of instructions addressed to the teacher, explaining which portions are to be read over by the student at home, how the laboratory is to be arranged, and how the work is to be distributed.

A new English edition[2] of Kohlrausch's *Physical Measurements* is a welcome addition to our laboratory literature. The translation from the second German edition which has been doing duty up to the present time as the American Kohlrausch was exceedingly incomplete, being only half the size of the present volume. In addition it was marred by numerous mistakes, and poorly printed. The arrangement of the four-place logarithms, printed on both sides of the same leaf, and so far from the back cover as to make them inconvenient to find, was sufficient to disgust the average physicist. Those who were not conversant with the German language preferred the English edition of 1883, even at the increased price, on account of the superior value. The want of a new edition has been felt.

In this new edition, the translation closely follows the German, and is

[1] *Practical Lessons in Physical Measurement.* By ALFRED EARL. pp. xv + 350. Macmillan & Co., 1894.

[2] *An Introduction to Physical Measurements.* By F. KOHLRAUSCH. Third English, translated from the seventh German edition, by T. H. WALLER and H. R. PROCTOR. pp. 476. D. Appleton & Co.

well done, the lack of sufficient cuts and of detail in the directions and descriptions being the fault of the original.

The book is replete with suggestions, covering all the best methods in use at the present time, and is particularly good in the enumeration of the sources of error attending the various operations. But it can hardly be properly called an "Introduction," presupposing, as it does, a good acquaintance with the subject. In such branches as the proper use of the balance, the calibration of thermometers, and parts of the optical work and magnetic determinations, it is unexcelled.

We have sought in vain for the "notes, appendices, and tables signed 'TR.,' added by the translators" mentioned in the preface.

HOLBROOK CUSHMAN.

A Treatise on the Measurement of Electrical Resistance. By WILLIAM A. PRICE, M.A. 8vo, pp. xvi+199. Oxford, The Clarendon Press, 1894.

The author of this book writes from knowledge acquired by long practical experience in the construction of resistance coils, and the work contains a large amount of useful information, and precise directions relating to the winding and mounting of resistance coils, including standards. While it cannot be said that a large part of this information is not to be found elsewhere, yet it is here assembled in a very satisfactory and accessible form.

In the chapter on materials for resistance coils the author gives a succinct account of the general principles, derived from Matthiessen's investigations, which should govern the choice of an alloy for purposes of resistance. The wide divergence of different samples of German silver wire is recognized, and attention is drawn to the brittleness or rottenness which it sometimes exhibits, particularly when exposed to corroding gases, or when frequently heated by an electric current. This undesirable property is probably due to the zinc constituent of the alloy. A single section only is devoted to the alloy platinoid, and no new light is thrown on the behavior of either this alloy or of the newer one known as manganin. A *resumé* is given of Dr. Lindeck's paper on that subject before the British Association in 1892. It is well known that the experts of the Berlin Reichsanstalt direct that manganin must be "artificially aged," by subjecting it to a temperature of 140° C. for five hours after it has been wound on its bobbin, and heavily coated with shellac varnish. It is altogether likely that other alloys intended for resistance coils would be improved by similar treatment. The resistance of a platinum-silver standard coil is lowered by having its temperature only moderately raised for an hour or so.

The Wheatstone's bridge is very satisfactorily treated in a separate chapter which leaves little to be desired. Much useful information on the slide wire bridge will be found in the succeeding chapter. The only contact device described is defective in the very important point that all the pressure which the operator applies to the knob is transmitted to the bridge wire, which may in this way be indented and injured. A device is easily made in which the contact pressure is determined entirely by the tension of a small spiral spring.

Among the methods of comparing resistances, one is surprised not to find that best one of all — the Carey Foster method. This omission is the more remarkable because the method is of English origin and is used at the Cavendish laboratory for the comparison of the B. A. standards.

Three methods of measuring the resistances of batteries and electrolytes are quite fully described. Two of these depend upon the electromotive force of the battery under test. Since perceptible polarization takes place with an external resistance as high as 10,000 ohms, and with low resistance in a small fraction of a second, any method which is vitiated by polarization is inapplicable to cells which are incompetent to maintain a uniform continuous current. The condenser method is applicable to all kinds of batteries and is readily applied, but no mention is made of it. The theory of Mance's and Beetz' method "rests on the assumption that the values of the electromotive force, and the resistance of a battery, are independent of the current passing through it." But the internal resistance of a battery has a definite relation to the current. This fact is exemplified in a striking manner by the resistance of the best "dry cells." In a dry cell recently tested by the condenser method with some special appliances, with an external resistance of 80 ohms and a current of 0.0177 ampere, the internal resistance was 3.32 ohms; while with an external resistance of one ohm and a current of 0.921 ampere the internal resistance was only 0.61 ohm. The intermediate values lie on a smooth curve, apparently an hyperbola, when currents and internal resistances are plotted as coördinates. All that can be affirmed of the internal resistance of a battery is that it is the ratio between the potential difference between the terminals and the particular current flowing, or $rI = e$. Both r and e are functions of the current.

It is unfortunate that the author should not have made himself familiar with the practical details of determining the unit of resistance in absolute measure, in order that he might have made his book still more valuable by adding a plain account of one or two of the best methods. It would have been much more to the point than the mathematical appendices on the Wheatstone's bridge and the theory of Mance's method, which have little more to commend them than an "academical interest."

HENRY S. CARHART.

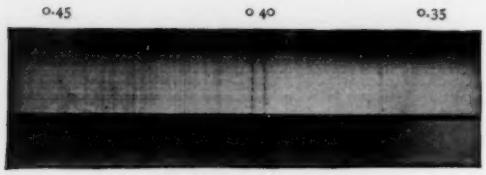

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

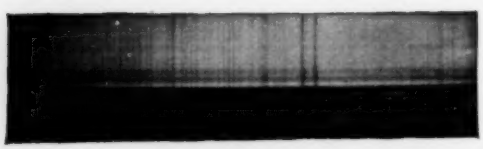

Fig. 5.

ERNEST NICHOLS: ULTRA VIOLET ABSORPTION.

UNIV. OF MICH.

To face the plate at p. 305. PHYSICAL REVIEW No. 10.

ERRATA.

The numbers indicating wave-lengths in Figs. 1 and 3 are incorrect. They should increase from left to right, instead of from right to left, the division at the left end of each figure being 0.33 μ. The value of each division of the scale is 0.01 μ. As a further check it may be remembered that the prominent double line (H) in each figure should lie just to the left of the division for 0.40 μ.